Lecture 3

Business problem (see section webpage after)

a) \[ q(p) = ap + b \]

\[ q(200) = 5000 \]

\[ \text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{-50}{1} = -50 \]
Here: \( a = -50 \),

So: \( q(p) = -50p + b \)

Plug in: \( q(200) = -50 \cdot 200 + b \)

\[ = 5000 \]

\[ b = 5000 \]

\( q(p) = -50p + 15000 \) (*)

b) \( C = C_{\text{fixed}} + C_{\text{variable}} \)

\[ = 100000 + C_{\text{variable}} \]

\( C(q) = 100000 + 75q \)
(c) \( R = pq \)

Want \( R = R(q) \)

So: find \( p \) in terms of \( q \),

use (*):

\[
q = -50p + 15000
\]

\[
50p = 15000 - q
\]

\[
p = 300 - \frac{1}{50}q
\]

So \( R = p \cdot q = (300 - \frac{1}{50}q) \cdot q \)

\[
= -\frac{1}{50} q^2 + 300q
\]
e) \( C(q) = 100000 + 75q \)

\[
\begin{align*}
& \text{(note: } q = 15000 \Rightarrow 300 - \frac{1}{50} q = 0 \Rightarrow R = 0) \\
\text{Two points of intersection} \\
\Rightarrow \text{two break even points.}
\end{align*}
\]

d) Solve \( C(q) = R(q) \):

\[
100000 + 75q = -\frac{1}{50}q^2 + 300q
\]
So: \( \frac{1}{50} q^2 - 225q + 100000 = 0 \)

Solve this quadratic for \( q \):

\[ q \approx 463, 10800 \]

4) \( P(q) = R(q) - C(q) \)

\[-\frac{1}{50} q^2 + 300q \]

\[-(75q + 100000)\]

\[-\frac{1}{50} q^2 + 225q - 100000\]
h) \[ P(q) = -\frac{1}{50} q^2 + 225q - 10000 \]

Profit maximized

Corresponds to: \( (q_1, P(q_1)) \) is vertex of parabola.
1) Midpoint of break even points

2) Complete the Square

\[ -\frac{1}{50} a^2 + 225a - 100000 \]

\[ = -\frac{1}{50} \left( a^2 - 11250a \right) - 100000 \]

\[ = -\frac{1}{50} \left( \left( a - 5625 \right)^2 - 5625^2 \right) - 100000 \]

\[ \left( x + y \right)^2 = x^2 + 2xy + y^2 \]

So: \[ a = 5625 \]
\[ P(q) = - \frac{1}{50} q^2 + 225q = 100000 \]

\[ P'(q) = - \frac{2}{50} q + 225 = 0 \]

\[ q = 56.25 \]

Calculus solution.
Introduction to limits

Take a ball, drop from 100 m: $s(t) = 100 - 5t^2$ (approx.)

position after $t$ seconds

$t = 0: s(0) = 100$
$t = 1: s(1) = 95$
$t = 2: s(2) = 80$
$t = 3: s(3) = 55$
Average velocity:

\[ V_{\text{avg}} (t=1, t=2) = \frac{s(2) - s(1)}{2 - 1} \]

\[ (\text{distance} \over \text{time}) \]

\[ = \frac{80 - 95}{2 - 1} = -15 \text{ (m/s)} \]

For any two times, this will find average velocity.
Instantaneous Velocity

\[ s \]

\[ \Delta t \]

\[ V_{\text{avg}}(t=1, t=1+\Delta t) = -15 \]

Want \( v(t) \): approximate via

\[ V_{\text{avg}}(t=1, t=1+\Delta t) \]

Take small \( \Delta t \)!