Webwork 1 is out, due this Sunday.
Homework 1 will be out Friday.

Lecture 4

Recall: \( z = x^2 + y^2 \)

\[ z = x^2 - y \]

Q: What are level curves of this?
have \( z_0 = x^2 - y \)

Write this as
\[
y = x^2 - z_0
\]

So: level curves are parabola
\[
y = x^2 \text{ shifted by } z_0
\]
Ex: \( z = f(x, y) = xy \)

Level curves: \( z_0 = xy \)

Then: \( y = \frac{x}{z_0} \) \( \left( x \neq 0 \right) \)

These are hyperbolas
\[ x = 0 : \quad z_0 = 0 \]

\[ 0 = x \cdot y \]

So: either \( x = 0 \) or \( y = 0 \)

\[ \uparrow \]

\[ y \text{-axis} \]

\[ \uparrow \]

\[ x \text{-axis} \]

0 level curve is just \( x \text{-axis} + y \text{-axis} \)
Partial Derivatives

\[ y = f(x) \]

then \( f'(x) \) measures how \( y \) changes as \( x \) changes.

Now: \( z = f(x, y) \), we can measure how \( z \) changes as we change only \( x \) or only \( y \).
Ex:

\[ f(x, y) \]

\[ y = b \]

Is \( f \) increasing at \((a, b)\)?

It depends

In \( x \)-direction: increasing
\nIn \( y \)-direction: decreasing
Suppose we fix $y = b$.

Then we obtain a curve through $f(a, b)$:

We can compute the derivative at $(a, b)$:

\[
\frac{df}{da}(a, b) = \lim_{h \to 0} \frac{f(a+h, b) - f(a, b)}{h}
\]
This the partial derivative of $f$ with respect to $x$ at $(a, b)$.

**Ex:** $f(x, y) = x^2 \cdot y$

$$
\frac{\partial}{\partial x} f(x, y) = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}
$$

$$
= \lim_{h \to 0} \frac{(x+h)^2 \cdot y - x^2 \cdot y}{h}

= \lim_{h \to 0} \frac{(x^2 + 2xh + h^2) \cdot y - x^2 \cdot y}{h}
$$
\[ \lim_{{h \to 0}} \frac{2x + 4h \cdot y}{h} = \lim_{{h \to 0}} (2x + 4y) = 2x y \]

\[ f(x, y) = x^2 y \]

\[ \frac{\partial}{\partial x} \left( x^2 y \right) = 2xy \]

Note: \( \frac{d}{dx} (x^2) = 2x \)
Observation:

To calculate the $x$-partial derivative, just of $y$ as a constant and differentiate "normally" (as in one variable calculus).

Summary

\[ \frac{\partial f}{\partial x} (x, y) = \frac{\partial}{\partial x} \left( f(x, y) \right) = f_x (x, y) \]

\[ = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h} \]

\[ \frac{\partial f}{\partial y} (x, y) = \frac{\partial}{\partial y} \left( f(x, y) \right) = f_y (x, y) \]

\[ = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h} \]
Ex: \( f(x, y) = \sin(x - y) \)

\[ \frac{\partial}{\partial x} (x, y) = \cos(x - y) \cdot y \]

Y constant for this

\[ \left( \sin(x - 2\pi) \right)' = \cos(2\pi) \cdot 2 \]

\[ \cos(2\pi) = 1 \]

Ex:

\[ f(x, y) = x^2 e^{xy} \]

\[ \frac{\partial}{\partial x} (x, y) = 2x \cdot e^{xy} + x^2 \left[ e^{xy} \right] \]
\[ a_y(x, y) = x^2 - x \cdot e^{xy} \]

\[ = x^3 e^{xy} \]