1. Let \( X \) denote the \( \Delta \) complex obtained as follows. Start with a standard 4-simplex, \( \Delta^4 \) with vertices 0, 1, 2, 3, 4. Consider the \( \Delta \) complex \( \partial \Delta^4 \) consisting of all 3-dimensional faces of \( \Delta^4 \). This \( \Delta \) complex has 5 3-dimensional faces, 10 2-dimensional faces, 10 1-dimensional faces and 5 0-dimensional faces. Form \( X \) from \( \partial \Delta^4 \) by identifying each 2-face \([abc]\) with the 2-face \([(4−c)(4−b)(4−a)]\) in the order-preserving way. Calculate the simplicial homology of \( X \).

2. Let \( X \) be the space obtained from \( \mathbb{R}^n \times \{0,1\} \) by identifying all points \((x,0)\) with \((x,1)\) whenever \( x \neq 0 \). That is, \( X \) is a version of \( n \)-space with a doubled origin. Let \( U \) denote the image of \( \mathbb{R}^n \times \{0\} \) in \( X \). Calculate the relative singular homology \( H_* (X,U;\mathbb{Z}) \). Determine whether or not it agrees with the reduced homology \( \tilde{H}_* (X/U;\mathbb{Z}) \).

3. Consider the solid cube with vertices \((a,b,c)\in\mathbb{R}^3\), where \(|a|=|b|=|c|=1\). It is a CW complex in an evident way. Identify opposite faces with a right-hand twist in the following fashion. Suppose \( F_1, F_2 \) are opposite faces and \( \vec{v} \) is the vector from the midpoint of \( F_1 \) to the midpoint of \( F_2 \). Then \( F_1 + \vec{v} \) coincides \( F_2 \), and if we rotate it by \( \pi/2 \) in either direction, it coincides with \( F_2 \) again. Identify the two faces by choosing the direction of rotation given by the curled fingers of your right hand when your thumb points along \( \vec{v} \).

Let \( X \) be the quotient space of the solid cube by these identifications. It is a CW complex, with the structure induced from that on the cube. Calculate its cellular homology.

4. Do question 30 from Section 2.2 of Hatcher. You may refer to the relevant section of the textbook in your answer.

5. Using the long exact coefficient sequence, show that if \( H_1 (X;\mathbb{Z}) = \mathbb{Q} \), then \( H_1 (X;\mathbb{Z}/(n\mathbb{Z})) = 0 \) for all \( n \).

Does there exist a CW complex \( X \) such that \( H_1 (X;F) = 0 \) whenever \( F \) is a field, but such that \( H_1 (X;\mathbb{Z}) \neq 0 \)?