1. Suppose \( f(z) \) is a complex polynomial of degree \( n \), viewed as a map \( f : \mathbb{C} \rightarrow \mathbb{C} \). Extend \( f \) to a map \( \hat{f} : S^2 \rightarrow S^2 \), where \( S^2 \) is the one-point compactification. Show that \( \hat{f} \) is homotopic to the map \( \hat{g} : S^2 \rightarrow S^2 \) induced by \( z \mapsto z^n \), and that this map is of degree \( n \).

2. Hatcher: chapter 2.2, question 11. For your convenience, this question is restated here.

A 3–dimensional CW complex is described as follows. A solid cube, \( C \), is given with the evident CW structure, consisting of 8 vertices (0–cells), 12 edges (closed 1–cells), 6 faces (closed 2–cells) and a single 3–cell. The following identifications are made: each face (closed 2–cell) is identified with the opposite face by means of a clockwise quarter-twist. That is, if \( F_1 \) and \( F_2 \) are two opposite faces of the cube, a homeomorphism \( \phi_{12} : F_1 \rightarrow F_2 \) is given by looking at \( F_1 \)—setting it to be the front face of the cube—, rotating it by \( \pi/2 \) in the clockwise direction around its midpoint, and then projecting it backwards onto the back face of the cube—\( F_2 \). One does this for each opposing pair of faces, giving \( \phi_{12}, \phi_{34} \) and \( \phi_{56} \). One now lets \( X \) denote the quotient of \( C \) by the relation \( x \sim \phi(x) \) where \( \phi \) denotes any of the three homeomorphisms given. The space \( X \) is still a CW complex, but has 1 3–cell and only 3 faces. Certain edges of the original cube have also been identified—the precise number of edges of \( X \) is not given here.

Write down a chain complex for the cellular homology of \( X \) and calculate \( H_*(X; \mathbb{Z}) \).


4. Fix a natural number \( n \). Construct the Moore space \( M(n, 1) \) as follows. Let \( f_n : S^1 \rightarrow S^1 \) be a degree-\( n \) map and let \( M(n, 1) \) be the CW complex having 1-skeleton \( S^1 \) and a single 2-cell, attached to \( S^1 \) by means of \( f_n \).

(a) Write down the homology \( H_*(M(n, 1); \mathbb{Z}) \).
(b) Write down the homology \( H_*(S^2 \times M(n, 1); \mathbb{Z}) \).
(c) Write down the homology \( H_*(M(n, 1) \times M(n, 1), \mathbb{Z}) \).
(d) Consider the map \( \phi : M(n, 1) \rightarrow S^2 \) given by collapsing the 1-skeleton of \( M(n, 1) \) to the base-point. This induces a map \( \phi \times \text{id} : M(n, 1) \times M(n, 1) \rightarrow S^2 \times M(n, 1) \). What is the induced map on integer-valued homology?

5. Does there exist a space \( X \) such that \( H_1(X; \mathbb{Z}) \neq 0 \) but \( H_1(X; F) \equiv 0 \) for all fields \( F \) (it suffices to check when \( F = \mathbb{Z}/(p) \) and \( F = \mathbb{Q} \))?