Math 527/427
Homework 2
28 January 2018

1. Prove the "5-lemma". That is, suppose given the following commutative diagram of R-modules

$$\begin{tikzcd}
A \arrow[r, "f"] \arrow[d, "\phi_a"] & B \arrow[r, "g"] \arrow[d, "\phi_b"] & C \arrow[r, "h"] \arrow[d, "\phi_c"] & D \arrow[r, "j"] \arrow[d, "\phi_d"] & E \arrow[d, "\phi_d"] \\
A' \arrow[r, "f'"] & B' \arrow[r, "g'"] & C' \arrow[r, "h'"] & D' \arrow[r, "j'"] & E'
\end{tikzcd}$$

   in which both rows are exact and where ϕ_a is surjective, ϕ_b is an isomorphism, ϕ_d is an isomorphism and ϕ_e is injective, then prove ϕ_c is an isomorphism.

2. Let p be a prime number. Determine all isomorphism classes of abelian groups A that can appear as the middle term of a short exact sequence

$$0 \to \mathbb{Z}/(p^a) \to A \to \mathbb{Z}/(p^b) \to 0$$

3. For each $d \in \mathbb{Z}$ and each $n \in \{1, \ldots\}$, describe a surjective map $S^n \to S^n$ of degree d—pay particular attention to $d = 0$.

4. Given a polynomial function $f : \mathbb{C} \to \mathbb{C}$, one may extend f to a function of the Riemann sphere $\hat{f} : S^2 \to S^2$, i.e., the one-point compactification of $\mathbb{C}$ to itself. Prove that the degree of $\hat{f}$ as a map of spheres agrees with the degree of f as a polynomial.

5. Give an example—with proof—of a topological space X that is homotopy equivalent to a CW complex, but that cannot be given a CW structure.