1. Prove the “5-lemma”. That is, suppose given the following commutative diagram of $R$-modules

\[
\begin{array}{ccccccccc}
A & \xrightarrow{f} & B & \xrightarrow{g} & C & \xrightarrow{h} & D & \xrightarrow{j} & E \\
\downarrow{\phi_a} & & \downarrow{\phi_b} & & \downarrow{\phi_c} & & \downarrow{\phi_d} & & \downarrow{\phi_d} \\
A' & \xrightarrow{f'} & B' & \xrightarrow{g'} & C' & \xrightarrow{h'} & D' & \xrightarrow{j'} & E'
\end{array}
\]

in which both rows are exact and where $\phi_a$ is surjective, $\phi_b$ is an isomorphism, $\phi_d$ is an isomorphism and $\phi_e$ is injective, then prove $\phi_c$ is an isomorphism.

2. Let $p$ be a prime number. Determine all isomorphism classes of abelian groups $A$ that can appear as the middle term of a short exact sequence

\[0 \to \mathbb{Z}/(p^a) \to A \to \mathbb{Z}/(p^b) \to 0\]

3. For each $d \in \mathbb{Z}$ and each $n \in \{1, \ldots\}$, describe a surjective map $S^n \to S^n$ of degree $d$—pay particular attention to $d = 0$.

4. Given a polynomial function $f : \mathbb{C} \to \mathbb{C}$, one may extend $f$ to a function of the Riemann sphere $\hat{f} : S^2 \to S^2$, i.e., the one-point compactification of $\mathbb{C}$ to itself. Prove that the degree of $\hat{f}$ as a map of spheres agrees with the degree of $f$ as a polynomial.

5. Give an example—with proof—of a topological space $X$ that is homotopy equivalent to a CW complex, but that cannot be given a CW structure.