1. Let $X = \{g,s\}$, and endow $X$ with the following topology: The subsets $\{\emptyset,X,\{g\}\}$ are open. Give $[0,1]$ the usual metric topology.

   (a) Suppose $f : X \to [0,1]$ is a continuous function such that $f(s) = 0$. Show that $f(g) = 0$.

   (b) Produce, with proof, a nonconstant continuous function $f : [0,1] \to X$.

2. Let $X$ be a topological space and let $A,B$ be two closed subsets of $X$ such that $X = A \cup B$. Let $Y$ be a topological space. Suppose $f : X \to Y$ is a function such that the restrictions $f|_A : A \to Y$ and $f|_B : B \to Y$ are continuous ($A$ and $B$ are given the subspace topologies). Prove that $f$ is continuous.

3. Let $(X,d)$ be a metric space. Recall that a sequence $(x_n)$ in $X$ is said to be a Cauchy sequence if, for all $\epsilon > 0$, there exists some $N_\epsilon \in \mathbb{N}$ such that $d(x_n,x_m) < \epsilon$ for all $n,m > N_\epsilon$. The space $X$ is said to be complete if every Cauchy sequence converges in $X$. Given an example of a homeomorphism $f : X \to Y$ of metric spaces where $X$ is complete and $Y$ is not complete.