1. Use the summation formulae for sines and cosines

\[
\sin(\phi + \theta) = \sin \phi \cos \theta + \cos \phi \sin \theta, \quad \cos(\phi + \theta) = \cos \phi \cos \theta - \sin \phi \sin \theta
\]

to express each of the following as a sum of powers of \( \sin \theta \) and \( \cos \theta \):

(a) \( \sin(2\theta) \),
(b) \( \sin(3\theta) \),
(c) \( \frac{2 \tan \theta}{1 + \tan^2 \theta} \).

2. In the following diagram

the triangle \( ABC \) is equilateral with side length 1, the angle \( \angle BAC \) is therefore \( \pi/3 \). The line \( AD \)
is a perpendicular bisector of \( BC \) and the curved line \( BC \) is the arc of the circle with centre \( A \) and radius 1. Call the arc \( \gamma \). The length of \( \gamma \) is \( \pi/3 \).

(a) Using the fact that a line segment is the shortest path between two points (you do not need to prove this), prove that \( 3 < \pi \).

(b) It was asserted in class that the length of \( \gamma \) is bounded above by \( |BC| + 2|DE| \), i.e. \( \pi/3 \leq |BC| + 2|DE| \). The proof of this fact requires integration, so it is not covered in this course. We will assume it. Using this fact, give a proof that \( \pi < 4 \). (Hint: to calculate \( |DE| \), first calculate \( |AD| \).)
(c) Calculate $|BE|$.

(d) Use the result for the last part to obtain a better lower bound for $\pi$ than 3.

3. The following identity may be proved by repeated use of the summation formula, among other methods:

$$\sin(5\theta) = \sin^5 \theta - 10 \sin^3 \theta \cos^2 \theta + 5 \sin \theta \cos^4 \theta$$

(a) Rewrite this in terms of $\sin \theta$ only (no $\cos \theta$ terms).

(b) Suppose $\theta$ is an angle such that $\sin(5\theta) = 0$ but $\sin \theta \neq 0$. Using the formula from the previous part, write down a polynomial equation of the form $a x^4 + b x^2 + c = 0$ satisfied by $\sin(\theta)$.

(c) Solve this polynomial equation for $x$ (you should get 4 possible values for $x$).

(d) What is $\sin(2\pi/5)$? Give an exact answer (not a decimal approximation). Explain how you know this.

4. Suppose a triangle has sides of length 7, 5 and 3. Let $\theta$ denote the largest internal angle of this triangle. Use the law of cosines to calculate $\cos \theta$. What is $\theta$?

5. Suppose $ABC$ is a triangle with sides of the following lengths $|AB| = 10$, $|BC| = 17$ and $|CA| = 9$. What is $\cos \angle CAB$? What is the area of $ABC$?