

Assignment 8

1.

2. Consider the function

$$f(x) = \frac{x^3 + x^2 - 2x - 3}{x^2 - 3}.$$

Its first and second derivatives are given by

$$f'(x) = \frac{(x^2 - 1)(x^2 - 6)}{(x^2 - 3)^2}, \quad f''(x) = \frac{2x(x^2 + 9)^2}{(x^2 - 3)^3}.$$

(a) Find all x such that $f'(x) = 0$ or $f''(x)$ does not exist.

(b) Find all x where $f(x)$ is defined and such that $f'(x) = 0$ or $f''(x)$ does not exist.

(c) On which intervals is $f(x)$ increasing? On which intervals is $f(x)$ decreasing?

(d) On which intervals is $f(x)$ concave up? On which intervals is $f(x)$ concave down?

(e) Find the coordinates of all local extrema and the inflection points, Be sure to indicate which is which.

(f) Find any asymptotes of the function $f(x)$ and write their equations.

(g) Draw a rough sketch of the graph of $f(x)$. Accurately place all critical points and inflection points, indicate all asymptotes, and make sure your graph shows where $f(x)$ is increasing and decreasing and correctly shows its concavity.