Assignment 7

1. Consider a water tank shaped like an inverted right circular cone of height 1 metre and radius 1 m. Let $h(t)$ denote the height of the water level of the tank in metres with time t in days. Water flows into the tank at a constant rate of 10 litres/day (1 litre=$10^{-3}m^3$.) Water evaporates from the tank at a rate of $0.01A$ in m^3/day, where A is the area in m^2 of the water surface.

   (a) When $h = 0.2m$, how fast is the water level rising?

   (b) If the tank is left for a long time, will it overflow?

2. Liquid is being poured into a parabolic bowl at a constant rate of $60\pi\, cm^3/s$. The volume of the bowl is given by $V = \pi x^4/2$, where the equation of the parabola is $y = x^2$, and y is the height of the liquid in the bowl. Find the rate of increase of the height of the liquid in the bowl when the height is 10 centimetres.

3. At 1:00 p.m. ship A is 25 km due north of ship B. If ship A is sailing west at a rate of 16km/h and ship B is sailing south at 20km/h, find the rate at which the distance between the two ships is changing at 1:30 p.m. (Be sure to draw a diagram).

4. Determine all the critical points for the function a) $g(t) = t^{2/3}(2t-1)$, b) $R(x) = \frac{x^2+1}{x^2-x-6}$.

5. Suppose f is a function defined on an interval I . If $f'(x)$ is positive for all x in I what can you conclude about the behaviour of f on I?

   Let f be defined on an interval I with an interior point c. If f has a local maximum at c, which of the following statements are true? (a) $f'(c) > 0$, (b) $f'(c) < 0$. (c) $f'(c) = 0$. (d) None of the above. (e) c is a critical point.

   If f has a local maximum at c and $f'(c)$ exists then what can you conclude about $f'(c)$?.

   State whether the following is true or false. If c is a critical point of f, then f must have a local maximum or local minimum at c. Justify your answer.

   If $f''(x) > 0$ on an interval, what can you conclude about $f'(x)$ in that interval?

   Can you draw the graph of a function $f(x)$ defined on $[-1, 6]$ that has no absolute minimum (if so, sketch it below). What does the extreme value theorem have to say about this situation.

   Draw the graph of a function $f(x)$ defined for all real values of x that is continuous and has both an absolute minimum and an absolute maximum.