MAXIMA AND MINIMA

Definition: Let f be a function defined on an interval I containing c. Then f has an **absolute maximum value** on I at c if $f(c) \geq f(x)$ for every x in I.

QUESTION: What conditions must be met to ensure that a function has an absolute maximum value and an absolute minimum value on an interval?
ANS: The function must be a continuous function on a closed interval.

EXTEME VALUE THEOREM: A function f that is continuous on a closed interval has an absolute maximum value and an absolute minimum value on that interval.

LOCAL MAXIMUM AND MINIMUM VALUES: Suppose I is an interval on which f is defined and c is an interior point of I. If $f(c) \geq f(x)$ for all x in some open interval containing c, then $f(c)$ is a local maximum value of f. If $f(c) \leq f(x)$ for all x in some open interval containing c, then $f(c)$ is a local minimum value of f.

Theorem (Local Extreme Point Theorem:) If f has a local minimum or maximum value at c and $f'(c)$ exists, then $f'(c) = 0$.

Definition of Critical Point: An interior point c of the domain of f at which $f'(c) = 0$ or $f'(c)$ fails to exist is called a **critical point** of f.

PROCEDURE FOR LOCATING ABSOLUTE MAXIMUM AND MINIMUM VALUES:

- Assume that the function f is continuous on the closed interval $[a, b]$.

- Locate the critical points c in (a, b); these are the points where $f'(c) = 0$ of $f'(c)$ does not exist.

- The critical points are the candidates for absolute maxima and minima.

- Compute the value of f at the critical points and at the end points of $[a, b]$.

- The largest value of f from the above step gives the absolute maximum and the smallest value gives the absolute minimum.

- If the interval is an open interval, then absolute extreme values, if they exist, occur at interior points.