Here is an explanation as to why the change is in the opposite direction for price and revenue if it is elastic at p_0.

- p=price, q=quantity, Demand function is $f(p)$ **expressed in terms of** p.

- Elasticity of demand $E(p) = \frac{-pf'(p)}{f(p}$.

Demand is elastic at some price p_0 if $E(p) > 1$, or what is the same, if $(1 - E(p_0)) < 0$.

Demand is inelastic at some price p_0 if $E(p_0) < 1$, or what is the same, if $(1 - E(p_0) > 0)$.

$R(p)$ = Revenue function **expressed in terms of price**, i.e. **as a function of** p.

$$R(p) = f(p).p.$$

By the product rule,

$$R'(p) = f(p) + p.f'(p) = f(p)\left(1 + \frac{pf'(p)}{f(p)}\right) = f(p)(1 - E(p)).$$

If demand is **elastic** at some price p_0, then, as $f(p)$ is always positive, we have

$E(p_0) > 1 \iff (1 - E(p_0))$ is negative $\implies R'(p_0) < 0 \implies R(p)$ is decreasing at p_0.

Note also that for some p_0, if $E(p_0) = 1$, then $R'(p_0) = f(p_0)(1 - E(p_0)) = 0$. So when you look at the **graph of the Revenue function as a function of the price** p, it means that the slope is zero at p_0 if $E(p_0) = 1$; hence the graph (which is an inverted parabola) has **maximum value at** p_0, which means that the revenue is maximum at p_0. We will use this later while we do **PROFIT MAXIMISATION** .

MORE PROBLEMS ON EXPONENTIAL GROWTH:

This is in continuation with the first problem on oil consumption. **You should always integrate over the required period when you are asked to find something over a period of time.** If you just substituted a value for t in the formula, this will only give you the amount consumed at that point of time t. In the oil consumption problem, note that **we were asked to find the amount of oil consumed over the period of a year** and NOT how much oil was being consumed at the end of one year. Recall that we computed the function as

$$y = 1.2e^{\ln}(1.015)t.$$

b) Find the function that gives the amount of oil consumed between $t = 0$ and a future time t.

Ans: So we have to integrate between time $s = 0$ and a future time $s = t$. **I have changed the variable here to s because the future time is denoted by t.** Thus the answer is

$$\int_0^t 1.2e^{\ln(1.015)s}\,ds = \frac{1.2}{\ln(1.015)} \times (e^{\ln(1.015)s}\,|_0^t = \frac{1.2}{\ln(1.015)} \times (e^{\ln(1.015)t} - 1) \text{ million barrels.}$$

c) After how many years will the amount of oil consumed reach 10 million barrels?

Ans: We want $\frac{1.2}{\ln(1.015)} \times (e^{\ln}(1.015)t - 1) = 10$. Thus

$$\begin{aligned} e^{\ln(1.015)t} &= 10 \times \tfrac{\ln(1.015)}{1.2} + 1 \\ (1.015)^t &= 10 \times \tfrac{\ln(1.015)}{1.2} + 1 \\ t \times \ln(1.015) &= 10 \times \tfrac{\ln(1.015)}{1.2} + 1 \\ t &= \tfrac{\ln((10\ln(1.015)/2)+1)}{\ln(1.015)} \end{aligned}$$

which gives $t \approx 7.85551$ years.

PROBLEM: One thousand dollars is invested at 5% interest compounded continuously. (a) Give the formula for the amount A as a function of time t, after t years.

ANS: See the notes sent yesterday; we have $P = 1000$, $r = 0.05$. Hence

$$A(t) = 1000 \times e^{0.05t}.$$

(b) How much money will be in the account after 6 years?

ANS: Here we just substitute as we are asked to find the value at the end of 6 years, so we substitute $t = 6$ to get

$$A(6) = 1000 \times e^{0.05\times 6} = 1000 \times e^{0,3} \approx \$1349.86.$$

(c) After six years, at what rate will $A(t)$ be growing?

ANS: Need to measure **RATE OF GROWTH**, which is the derivative $A'(t)$.

$$A'(t) = 1000 \times (0.05) \times e^{0.05t} = 50e^{0.05}t.$$

$$A'(6) \approx \$67.49.$$

Therefore after six years, the amount will be growing at the rate of \$67.49 per year.

(d) How long is it required for the initial investment to double?

ANS: We must find t so that $A(t) = \$2000$. So we look at $1000e^{.05t} = 2000$ and solve for t.

$$\begin{aligned}
1000\, e^{.05t} &= 2000 \\
e^{0.05t} &= 2 \\
\ln e^{.05t} &= \ln 2 \\
0.05t &= \ln 2 \\
t &= \frac{\ln 2}{.05} \approx 13.86 \text{ years.}
\end{aligned}$$

Therefore the amount will double after approximately 13.86 years.