# Practice Problems

1) a) Find the equation of the tangent line to the curve $f(x) = \frac{1}{\sqrt[3]{x^2}}$ that is parallel to ~~at~~ the line $y - 2x = \pi$.

Ans: $y = 2x + \pi$

$f(x) = x^{-2/3}$ ; $f'(x) = -\frac{2}{3} x^{-5/3}$

If the tangent line is parallel to the line $y = 2x+\pi$,

slope of the tangent line = slope of $y = 2x+\pi = 2$.

So $f'(x) = 2 \iff -\frac{2}{3} x^{-5/3} = 2$

$\iff x^{-5/3} = -3 \iff x = -3^{-3/5}$

$y = (3^{-3/5})^{2/3} = -3^{2/5}$

So the tangent line has slope 2 and passes through $(x, y) = (-3^{-3/5}, -3^{2/5})$

Equation of the line: $(y + 3^{2/5}) = 2(x + 3^{-3/5})$.

b) Find the equation of the line normal to the tangent line:

Ans: Line normal to the tangent line has slope $(-\frac{1}{m})$, where m = slope of tangent line. (Two lines are parallel if their slopes are equal. Two lines are perpendicular if the product of their slopes is $-1$. So the line normal to the tangent has slope $-\frac{1}{2}$ and passes through $(-3^{-3/5}, -3^{2/5})$, so the equation is

$$(y + 3^{2/5}) = -\frac{1}{2}(x + 3^{-3/5}).$$

Business problem:

~~a)~~ When EZ Electronics Company sells surge protectors at ~~\$500~~ \$50 a piece, they produce and sell 3000 of them per month. For every \$1 increase in price, the number of surge protectors they sell decreases by 15. Assume that the fixed production costs are \$50,000 and the variable costs are \$30 per surge protector.

a) Find the linear demand function $q = D(p)$, where $p$ is the price of a unit and $q$ is the number of surge protectors made and sold. [Hint: The point $(p, q) = (50, 3000)$ must lie on this line).

Ans: $\dfrac{q - 3000}{p - 50} = -15$

$\Rightarrow$ Demand equation is

$$p = -\frac{1}{15}q + 250.$$

Let $p = mq + c$ be the linear demand equation.

when $p = 50$, $q = 3000$

when $p = 51$, $q = 3000 - 15 = 2985$

$50 = mq + c \Rightarrow 50 = 3000m + c$

& $51 = 2985m + c$

$\Rightarrow 1 = -15m$, $m = -\frac{1}{15}$

and $c = 250$.

b) Find the Cost function $C(q)$ and express it as a function of $p$.

Ans: $C(q) = 50000 + 30q$

$C(p) = 50000 + 30(-15p + 3750)$

$= -450p + 162500.$

c) Find the revenue function $R(q)$ as a function of $q$, and then express it as a function of $p$.

Ans: $R(q) = p(q) \times q = -\frac{1}{15}q^2 + 250q.$

$R(p) = p \cdot q(p) = -15p^2 + 3750p$ (Note: $q = 250 - 15p$)

d) Find the marginal profit, $MP(p)$, with respect to $p$.

Ans. $P'(p) = R'(p) - C'(p)$

$= -30p + 3750 + 450$

$= -30p + 4200.$

(e) Find the price at the break-even points. You may leave your answer undimplified.

Ans: $R(p) = C(p)$

$\Rightarrow -15p^2 + 3750p = -450p + 162500$

$\Rightarrow 15p^2 - 4200p + 162500 = 0$

$\Rightarrow p = \dfrac{4200 \pm \sqrt{(4200)^2 - (4 \times 15 \times 162500)}}{30}.$

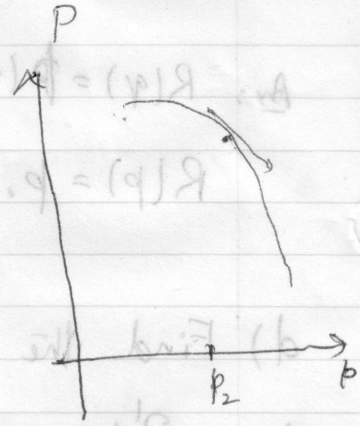
(f) If EZ electronics company is operating at the higher break-even point, should it increase or decrease the price of its surge protectors to increase its profits? Explain your answer.

Ans: Higher break-even point $P_2$ from part c) will satisfy

$P_2 > \frac{4200}{30}$.

From part d), $P'(P_2) = -30 < 0$.

(negative slope)




If price is decreased, profit increases.

Hence price should decrease from $P_2$ inorder to increase profit.

CAUTION: Be careful in plotting your curve and use the right variable ($p$ or $q$) depending on the questions.