Problems in Continuity and Differentiability; Derivatives as rate of change; Deriving functions

1. Find the number c that makes

$$f(x) = \begin{cases} \dfrac{x-c}{c+1}, & \text{if } x \leq 0 \\ x^2 + c, & \text{if } x > 0 \end{cases}$$

continuous for every x.

2. Find the values of a and b so that

$$f(x) = \begin{cases} ax + b, & \text{if } x < 0 \\ 2\sin(x) + 3\cos(x) & \text{if } x \geq 0 \end{cases}$$

is differentiable at $x = 0$.

3. The price p (in dollars) and the demand q (in thousands of units) of a commodity satisfy the demand equation $6p + q + qp = 94$. Find the rate at which demand is changing when $p = 9$, $q = 4$, and the price is rising at the rate of 2 per week.

4.When EZ Electronic Company sells surge protectors at \$50 a piece, they produce and sell 3000 of them per month. For every \$1 increase in price, the number of surge protectors they sell decreases by 15. Find the linear demand function $q = D(p)$, where p is a price of a unit and q is the number of surge protectors made and sold. [Hint: The point $(p, q) = (50, 3000)$ must lie on this line.]

5. Find the derivatives of the following functions using differential rules(product rule, quotient rule, etc.). DO NOT SIMPLIFY

(a) $\sqrt{\dfrac{\cos 2x}{\sin x}}$

(b)$(\sqrt{x}+x+2)((x+1)^3-2)$

(c)$y=\dfrac{e^{x^2-1}}{\sin(x^2)}$.

6. Use the definition of derivatives to find $f'(x)$ for $f(x)=\sqrt{x^2-1}$. NO CREDIT WILL BE GIVEN FOR USING OTHER METHOD.