Problems in Finding Derivatives and Tangent Lines SOLUTION

1. Use the definition of the derivative to compute $f'(1)$ for $f(x) = \dfrac{13}{x+7}$. NO CREDIT will be given for any other method.

Solution:

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$
$$= \lim_{h\to 0} \frac{\frac{13}{1+h+7} - \frac{13}{8}}{h}$$
$$= \lim_{h\to 0} \frac{13\cdot 8 - 13(8+h)}{8h(8+h)}$$
$$= \lim_{h\to 0} \frac{-13h}{8h(8+h)}$$
$$= \lim_{h\to 0} \frac{-13}{8(8+h)}$$
$$= \frac{-13}{64}$$

2. Compute the derivatives of the following functions. DO NOT SIMPLIFY.

(a) $f(x) = (sinx + x^2 + 1)(2x^3 + x)^2$

(b) $g(x) = \dfrac{x^2 + 12x + e^3}{x + e^x}$

(c) $h(t) = e^{3t}(t^2 + x^2)$

Solution:

(a) $f'(x) = (cosx + 2x)(2x^3 + x)^2 + 2(sinx + x^2 + 1)(2x^3 + x)(6x^2 + 1)$

(b) $g'(x) = \dfrac{(2x + 12)(x + e^x) - (x^2 + 12x + e^3)(1 + e^x)}{(x + e^x)^2}$

(c) $h'(t) = 3e^{3t}(t^2 + x^2) + e^{3t}(2t)$

3. Find $h'(1)$ where $h(x) = \dfrac{xg(x)+7}{f(x)}$, $f'(1) = 4$, $g'(1) = -2$, and $f(1) = 1$, $g(1) = 1$. EXPREE YOUR ANSWER AS AN INTEGER.

Solution:

$$h'(x) = \frac{f(x)(xg(x)+7)' - (xg(x)+7)f'(x)}{[f(x)]^2}$$

$$= \frac{f(x)[g(x)+xg'(x)] - [xg(x)+7]f'(x)}{[f(x)]^2}$$

$$\therefore h'(1) = \frac{f(1)[g(1)+g'(1)] - [g(1)+7]f'(1)}{[f(1)]^2}$$

$$= \frac{1[1+(-2)] - [1+7]4}{[1]^2}$$

$$= -33$$

4. Find the equation of the tangent line to $y = f(x) = \dfrac{x+3}{2x+1}$ at the point corresponding to $x = 0$.

Solution:

The slope of the tangent line at $x = 0$ is $f'(0)$.

$$f'(x) = \frac{(2x+1) - 2(x+3)}{(2x+1)^2}.$$

$$\therefore f'(0) = \frac{1-6}{1} = -5.$$

Therefore, the equation of the tangent line is: $y = -5x + b$.

At $x = 0$, $y = f(0) = \dfrac{3}{1} = 3$. Therefore $3 = 0 + b$, i.e. $b = 3$.

The equation of the tangent line is: $y = -5x + 3$.

5. Find the y-intercept of the tangent line to the curve $y = x^3 + 1$ at the point $(2, 9)$.

Solution:

$y'(x) = 3x^2$.
At the point $(2, 9)$, $y'(2) = 3 \cdot 2^2 = 12$.
Therefore the equation of the tangent line is: $y = 12x + b$.
Hence, the y-intercept $b = y - 12x = 9 - 12 \cdot 2 = -15$.

6. Find the x and y coordinates of the point on the graph of $y = \frac{1}{4}(2x + 1)^2$ where the tangent line is parallel to the line $y - 3x = 1$.

Solution:

The slope of the tangent line of $y = \frac{1}{4}(2x + 1)^2$ is:

$$y'(x) = \frac{1}{4} \cdot 2(2x + 1) \cdot 2 = 2x + 1.$$

Since the tangent line is parallel to the line $y - 3x = 1$, the tangent line has a slope equal to 3.
Therefore $y'(x) = 2x + 1 = 3. \implies x = 1$.
$y = \frac{1}{4}(2 \cdot 1 + 1)^2 = \frac{9}{4}$.
The x, y coordinates are $x = 1$, $y = \frac{9}{4}$.