

## Problems in Finding Derivatives and Tangent Lines COMMENTS:

1. A common approach to solving this problem was to first find  $f'(x)$  and then substitute  $x = 1$  to find  $f'(1)$ . This is completely correct but requires more work than is necessary. A more efficient solution would be to simply find  $f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$ .

Remember also to keep  $\lim_{h \rightarrow 0}$  around until all necessary substitutions have been made. For those that first found  $f'(x)$ , it is important to distinguish  $h \rightarrow 0$  from  $x \rightarrow 0$ . Do not substitute  $x$  with 0.

2. Please keep the instructions in mind for this problem. After you differentiate, you *do not* need to make any simplifications to your answer.

(a) This part was very well done. Don't forget to apply the chain rule to  $(2x^3 + x)^2$ .

(b) This part was well done. Note that  $e^3$  is a constant so  $\frac{d}{dx}e^3 = 0$ .

(c) This question requires you to pay close attention to the fact that you differentiate with respect to  $t$ . You should assume that  $x$  is constant in this case so  $\frac{d}{dt}(t^2 + x^2) = 2t$ . A common but incorrect answer claimed that  $\frac{d}{dt}(t^2 + x^2) = 2t + 2x$ .

3. Most mistakes related to this problem were related to finding the derivative of  $xg(x) + 7$ . By the product rule,  $\frac{d}{dx}(xg(x) + 7) = g(x) + xg'(x)$ . A common mistake was to write  $\frac{d}{dx}(xg(x) + 7) = xg'(x)$ . This incorrectly assumes that  $x$  can be treated as a constant.

4. Question 4 was very well done. Recognizing that  $(2x + 1)^2$  in the denominator of  $f'(x)$  does not need to be expanded can save a little time.

5. This problem was generally well done. For those that were stuck, try finding the equation of the tangent line as in the previous question. The y-intercept should be clear from there.

6. There was a little confusion on how to approach this problem. The following are two important facts about parallel lines. You may wish to draw a few pictures to convince yourself that they are indeed true.

(1) If two lines are parallel, then they must have the same slope.

(2) Two lines that have the same slope are parallel.

The latter is useful here. We want to find the point on the graph that will give a tangent line with the same slope as the line  $y - 3x = 1$ .

Rearranging the equation of the line gives  $y = 3x + 1$ , which clearly has a slope of 3.

To find the slope of the tangent line to  $y = \frac{1}{4}(2x + 1)^2$ , we differentiate to obtain  $y'(x) = 2x + 1$ .  $y'(x)$  represents slope of  $y = \frac{1}{4}(2x + 1)^2$  at various values of  $x$ . Solve  $y'(x) = 3$  to find the value of  $x$  that corresponds to a slope of 3. The solution is  $x = 1$ .

Plugging  $x = 1$  back into the original equation  $y = \frac{1}{4}(2x + 1)^2$  gives  $y = \frac{9}{4}$ . The desired coordinates are therefore  $(x, y) = (1, \frac{9}{4})$ .