Intermediate Value Theorem WS:

1. Question 1 was very well done.

2. Question 2 was very well done. We interpret "Does IVT apply?" to mean whether or not we can apply IVT using the interval $[-1, 3]$ rather than any smaller interval like $[2, 3]$.

3. Question 3 is tricky. Notice that $-3 < f(-1) < -2$ and $-2 < f(3) < -1$. Hence you cannot apply the IVT on any value L that does not satisfy $-3 < L < -1$. In particular, you cannot apply it for $L = 0$ and $L = 4$.

4. Question 4 was very well done. Try comparing it to the function in the previous question.

5. Question 5 was very well done. You can draw any continuous function defined on $[-1, 3]$.

6. Please note that this question asks specifically about what the IVT tells you. It is useful to rewrite the theorem in a form where its hypotheses and conclusion are clear. My version is below.

   If both of the following conditions are satisified

   H1. f is defined and continuous on $[a, b]$

   H2. L is an intermediate value satisfying $f(a) < L < f(b)$ or $f(b) < L < f(a)$,

   then there exists $c \in (a, b)$ such that $f(c) = L$.

   We can now examine each question individually.

   i. This question asks about values of L that are not intermediate values. H2 is not true in this case so the IVT does not apply.

      A common mistake was to assume that the function is not continuous if it reaches non-intermediate values in the interval. See the graph of question 3 for a counterexample.

   ii. If f is not continuous, then it does not satisfy H1. Thus the IVT cannot make a conclusion.

      A common mistake was to assume that the function would not reach some intermediate value L in the interval. Try $f(x) = \tan x$ and take two points x_1 and x_2 separated by more than π (i.e. $|x_1 - x_2| > \pi$).

   iii. The IVT concludes that there exists a c with $a < c < b$ but does not say anything about the exact location of c within the interval.

   iv. The IVT says that c exists (but does not tell you that it must be unique!). Hence there is at least one c.

Limits and Continuity WS:

1. All parts of this question were generally very well done. The most common mistake was saying limit i, ii, or iv does not exist. Keep in mind that $\lim_{x\to a} f(x)$ does not necessarily have to equal $f(a)$ and that $\lim_{x\to 2^-} f(x)$ is not the same as $\lim_{x\to -2} f(x)$. Please see the definition of limits for further clarification.

2. Parts i-iii were very well done.

   In part iv, be wary of writing $(-\infty, 0) \cup [0, \infty)$. This is the same thing as $(-\infty, \infty) = \mathbb{R}$. You can instead say that the function is continuous on the intervals $(-\infty, 0)$ and $[0, \infty)$.

   A note on interval notation: write (a, ∞) instead of $(a, \infty]$. The latter is confusing since it seems to imply that ∞ is in the set (∞ is not a real number).