

# Review session problems

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December 3, 2013

1 Estimate the value of  $\sqrt{26}$  using linear approximation. Is this an underestimate or an overestimate? Use the information to determine an interval which you can be sure contains the exact value  $\sqrt{26}$ .

2 A steel company ABC steel manufactures nuts and bolts. When  $x$  nuts are produced, they can be sold for  $-3x + 500$  dollars each, When  $y$  bolts are produced, they can be sold for  $-y + 300$  dollars each. Assume that nuts and bolts weight 0.5 kg each. How many nuts anyhow many bolts must be produced to maximize the revenue from 100kg of steel? Justify your answer.

3 Consider the curve  $x^2 + y^3 - 2xy = 0$ . Assume that the point  $(x, y) = (1, 1)$  lies on the curve, and that nearby points on the curve satisfy  $y = f(x)$  for some function of  $f(x)$ . Find  $f(1)$ ,  $f'(1)$  and  $f''(1)$ . Approximate  $f(1.02)$  using the linear approximation of  $f(x)$  at  $x = 1$ . State whether you expect this to be an overestimate or an underestimate of  $f(1.02)$ .

(4) Consider the function  $f(x) = \frac{2x^3}{3x^2-9}$ . Its first and second derivatives are given by

$$f'(x) = \frac{2x^2(x^2 - 9)}{3(x^2 - 3)^2}, \quad f''(x) = \frac{4x(x^2 + 9)}{(x^2 - 3)^3}.$$

Find the intervals of increase and decrease. On which intervals is  $f(x)$  concave up (resp. concave down)? Find the  $x$ -coordinate of all local min, local max, and inflection points. Write down the horizontal and vertical asymptotes. Draw a rough sketch of the graph of

$f(x)$ .