Optimization problems

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Steps for solving Optimization problems:

1) Read the problem carefully, identify the variables, and organize the given information with a picture.

2) Identify the objective function (the function to be optimized). Write it in terms of the variables of the problem.

3) Identify the constraints. Write them in terms of the variables of the problem.

4) Use the constraints to eliminate all but one independent variable of the objective function.

5) With the objective function expressed in terms of a single variable, find the interval of interest for that variable.

6) Use methods of calculus to find the absolute maximum or minimum value of the objective function on the interval of interest. If necessary, check the end points.

Problems

1) A boat on the ocean is 4 miles from the nearest point on a straight shore line; that point is 6 miles from a restaurant on the shore. A woman plans to row the boat straight to a point on the shore and then walk along the shore to a restaurant.

a) If she walks at 3 miles /hr and rows at 2 miles/hr, at which point on the shore should she land to minimize the total travel time. b) If she walks at 3 miles/hr, what is the minimum speed at which she must row so that the quickest way to the restaurant is to row directly (with no walking).

2) A box-shaped shipping crate with a square base is designed to have a volume of 16 m^3. The material used to make the base costs twice as much per square metre as as the material in the sides, and the material used to make the top costs half as much per square metre as the material in the sides. Suppose the material to make the sides costs \$20 per square metre. What are the dimensions of the crate that minimize the cost of the materials?