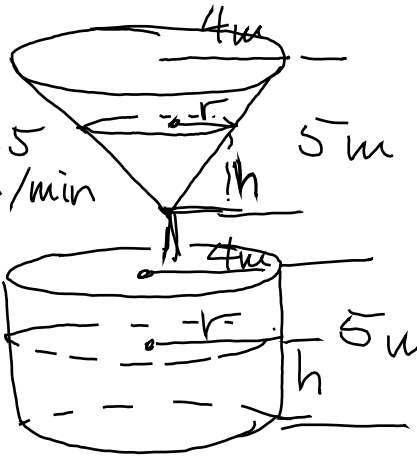
§3.10 Problem #38



$\frac{dh}{dt}_{top} = -0.5$ m/min

when $h_{top} = 3$m

$V_{top} = \frac{1}{3}\pi r^2 h$

$V_{bottom} = \pi r^2 h$

$$\frac{dV}{dt}_{top} = \text{outflow} = \text{inflow} = \frac{dV}{dt}_{bottom}$$

$$\frac{dV_{top}}{dt} = \frac{1}{3}\pi\left[2r\frac{dr}{dt}\cdot h + r^2\frac{dh}{dt}\right]$$

$$\frac{r}{h} = \frac{4}{5}$$

$$\Rightarrow r = \frac{4}{5}h$$

$$\Rightarrow \frac{dr}{dt} = \frac{4}{5}\frac{dh}{dt}$$

$$= \frac{1}{3}\pi\left[2\cdot\left(\frac{4}{5}h\right)\left(\frac{4}{5}\frac{dh}{dt}\right)\cdot h + \left(\frac{4}{5}h\right)^2\frac{dh}{dt}\right]$$

$$= \frac{1}{3}\pi\left[\frac{48}{25}h^2\frac{dh}{dt}\right]$$

At $h = 3$ m, we have $\frac{dh}{dt} = -0.5$ m/min

$$\frac{dV_{top}}{dt} = \frac{1}{3}\pi\left[\frac{48}{25}\cdot 3^2(-0.5)\right] = -\frac{24}{25}\pi \text{ m}^3/\text{min}$$

$$= -72/25\,\pi \text{ m}^3/\text{min}$$

Bottom.

$$\frac{dV}{dt}_{\text{bottom}} = -\frac{dV}{dt}_{\text{top}} = -\left(-\frac{72}{25}\pi\right) m^3/min$$

$$= \frac{72}{25}\pi \; m^3/min.$$

$$V_{\text{bottom}} = \pi r^2 h$$

$$= \pi (4)^2 h$$

$$= 16\pi h$$

$$\Rightarrow \frac{dV}{dt}_{\text{bottom}} = 16\pi \frac{dh}{dt}_{\text{bottom}}$$

$$\Rightarrow \frac{72}{25}\cancel{\pi} = 16\cancel{\pi} \frac{dh}{dt}_{\text{bottom}}$$

$$\Rightarrow \frac{dh}{dt}_{\text{bottom}} = \frac{72}{16 \cdot 25} = \frac{9}{50} \; m/min$$

$$= 0.18 \; m/min.$$