Assignment 5 Comments

1. Most students were able to figure out how to complete this problem but a number of mistakes were made in manipulating the equations.

   Please take a look at the approach used in the solution. Differentiating the equation twice implicitly allows one to avoid the quotient rule, which is nice because it makes computations easier and hence reduces the likelihood of mistakes.

2. Question 2 is similar to question 1; please refer to the notes above.

3. This question was very well done. Factoring the given expression gives a clever (shorter) solution to the problem.

4. This question was very well done. If you find $\frac{dy}{dx}$ at once using the given equation, remember to apply chain rule on the term inside the logarithm (i.e. $\frac{\mathrm{d}}{\mathrm{d}x} \ln y = \frac{y'}{y}$).

5. The computations in this question were very involved. Some approaches to this problem can lead to very complicated derivatives. To avoid unnecessarily difficult derivatives, it is worthwhile to check if any rearrangements can be made to your equation before differentiating.

   Several common mistakes are listed below:

   (1) $\dfrac{\mathrm{d}}{\mathrm{d}x} y^x \neq x y^{x-1}$. The formula $\dfrac{\mathrm{d}}{\mathrm{d}x} x^n = n x^{n-1}$ works only for constant n.

   (2) $\dfrac{\mathrm{d}}{\mathrm{d}x} y^x \neq y'(y^x \ln y)$. The formula $\dfrac{\mathrm{d}}{\mathrm{d}x} a^x = a^x \ln a$ works only for constant a.

   (3) We know $\ln(16x) = \ln x + \ln 16$ (why?) BUT $\ln(x+16) \neq \ln x + \ln 16$. Please review the properties of logarithms if this seems confusing.

   To address (1) and (2), here is my solution:

   $$\frac{\mathrm{d}}{\mathrm{d}x} y^x = \frac{\mathrm{d}}{\mathrm{d}x} e^{x \ln y} = e^{x \ln y} \cdot \frac{\mathrm{d}}{\mathrm{d}x}(x \ln y) = y^x \left(\ln y + \frac{x y'}{y} \right).$$

   By replacing y^x with $e^{x \ln y}$, we have made the base of the exponent constant. Applying chain rule and product rule gives the answer.