Does $f(x) = x^3 + 1$ take the value 1.5 for some x between -1 and 1?

$$f(-1) = 0, \quad f(1) = 1 + 1 = 2.$$

$f(x)$ continuous. IVT $\Rightarrow$ Yes, as 1.5 is between 0 and 2.

Eg: Finding an interest rate : Suppose you invest $\$1000$ in a special 5 year savings account with a fixed annual interest rate r, with monthly compounding. The amount of money A in the account after 5 years (60 months) is

$$A(r) = 1000 \left(1 + \frac{r}{12}\right)^{60}.$$ You wish to have $\$1400$ in the account after 5 years.

(a) Use the IVT to show that there is a value of r in $(0, 0.08)$ (i.e. interest rate between 0% and 8% for which $A(r) = 1400$.

(b) Use a graphing utility to illustrate your explanation in part (a) and then estimate the interest rate required to reach your goal.

Ans : (a) $A(r) = 1000 \left(1 + \frac{r}{12}\right)^{60}$ (continuous)

$A(0) = 1000, \quad A(0.08) = 1489.85$

$A(0) < 1400 < A(0.08)$

$\therefore \exists$ such an r.