# PRACTICE PROBLEMS.

~~Assignment 8~~

1. Sketch a complete graph of the function $f(x) = \frac{x^2}{x^2-4}$. You should cheek for asymptotes, local extrema and inflection points and mark these clearly on the graph. You may use the fact that $f''(x) = \frac{24x^2+32}{(x^2-4)^3}$.

$$f'(x) = \frac{2x(x^2-4) - 2x^3}{(x^2-4)^2} = \frac{2x(x^2-4-2x^2)}{(x^2-4)^2}$$

$$= \frac{-8x}{(x^2-4)^2}.$$

$f'(x)$ and $f(x)$ defined everywhere except at $x = \pm 2$.

Points to consider $x = \pm 2$.

Critical points of $f$: $f'(x)=0$ as $f(x)$ defined everyw

Hence $x = 0$ is the only critical point.

| | $x<-2$ | $x=-2$ | $-2<x<0$ | $x=0$ | $0<x<2$ | $x=2$ | $x>2$ |
|---|---|---|---|---|---|---|---|
| $f'$: | $+$ | | $+$ | | $-$ | | $-$ |
| $f$: | $\nearrow$ | $f$ undefined | $\nearrow$ | | $\searrow$ | | $\searrow$ |

2. Consider the demand equation

$$q = f(p) = \frac{1}{\sqrt{1+p}}.$$

(a)Find the elasticity of demand function $E(p)$.

(b) When $p = 10$, how will an increase in price affect the profit?

a) $\mathcal{E}(p) = \frac{p}{q} \times \frac{dq}{dp}$

$\frac{dq}{dp} = \frac{d}{dp}\left((1+p)^{-\frac{1}{2}}\right) = \frac{-1}{2}(1+p)^{-\frac{3}{2}}$

$\boxed{q = (1+p)^{-\frac{1}{2}}}$

$\mathcal{E}(p) = \frac{p}{(1+p)^{-\frac{1}{2}}} \times \frac{dq}{dp}$

$= \frac{p}{(1+p)^{-\frac{1}{2}}} \times -\frac{1}{2}(1+p)^{-\frac{3}{2}}$

$= -\frac{1}{2} p \times (1+p)^{-1} = \frac{-1}{2}\left(\frac{p}{1+p}\right)$

When $p = 10$, $\mathcal{E}(p) = \mathcal{E}(10) = -\frac{1}{2} \times \frac{10}{11}^{5} = -\frac{5}{11}$

$|\mathcal{E}(p)| < 1$, so price inelastic.

$p\uparrow \; R\uparrow$ ($R$ = Revenue)



Assuming Cost doesn't change, Profit = R − C, hence Profit ↑.

At $x=0$ : $f''(x)\Big|_{x=0} = \dfrac{24x^2+32}{(x^2-4)^3}\Bigg|_{x=0}$

$$= \frac{32}{(-4)^3} = \frac{\cancel{32}^{\,2}}{\underset{4}{\cancel{-64}}} = -\frac{1}{2}.$$

$f''(0)<0$, so $x=0$ is local max ; $f(0)=0$.

<u>Asymptote</u> : $\displaystyle\lim_{x\to\pm\infty} \frac{x^2}{x^2-4} = \lim_{x\to\pm\infty} \frac{x^2(1)}{x^2(1-4/x^2)} = 1.$

So $y=1$ is a horizontal asymptote.

$$f''(x) = \frac{24x^2+32}{(x^2-4)^3}$$

$f''(x)$ never zero, but $x=\pm 2$ are still points to consider.

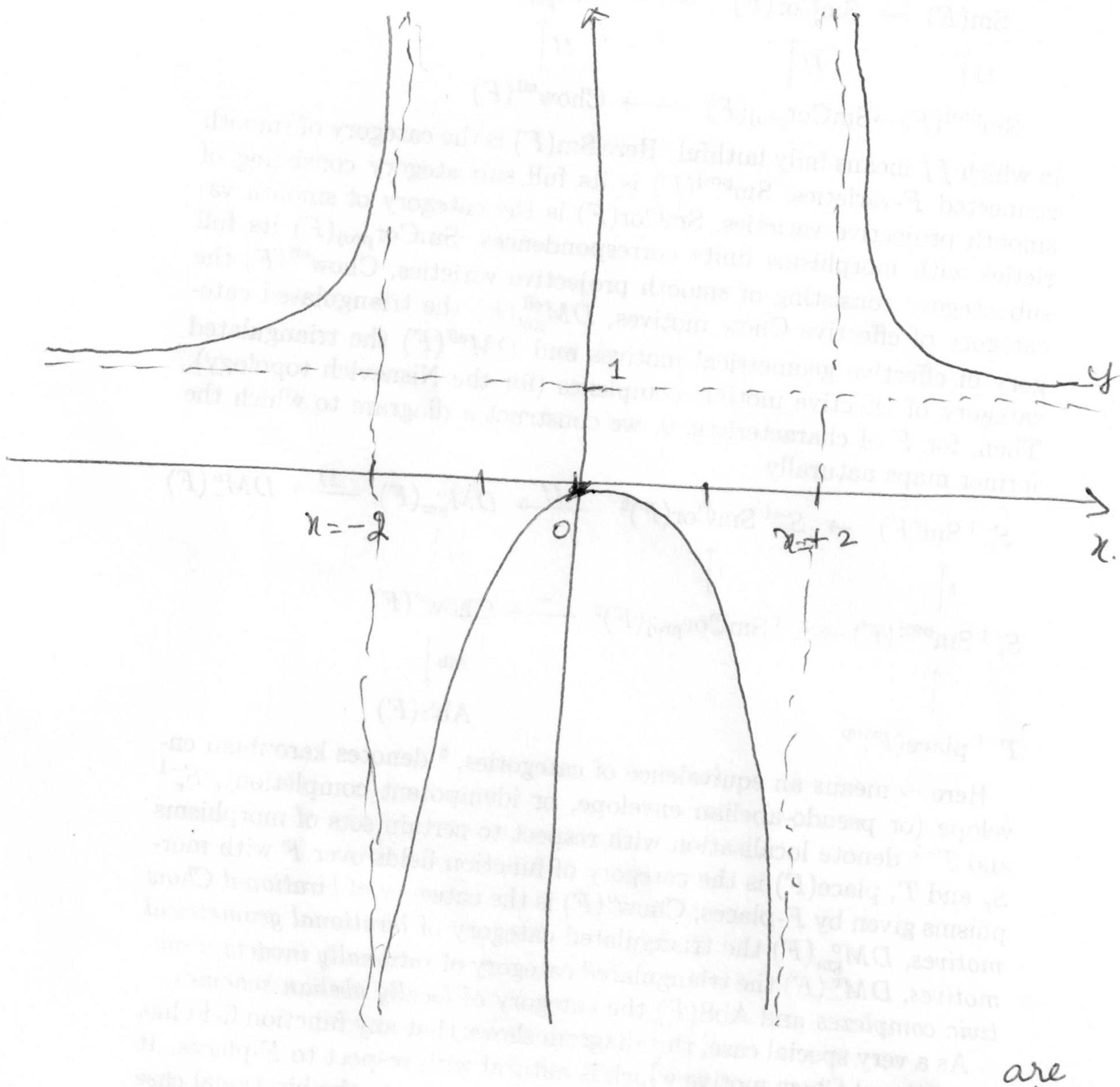
| | $x<2$ | $-2<x<2$ | $x>2$ |
|---|---|---|---|
| $f''$ | $>0$ | $<0$ | $>0$ |
| $f$ | | | |

$x=-2$ $\quad$ $x=2$

Asymptotes: Vertical asymptotes : $x = \pm 2$.

Intercepts : $x$-intercept : $f(x) = 0 \Rightarrow x = 0$ ;

$y$-intercept : $f(0) = 0$




Function is symmetric. Note $x = \pm 2$ are ~~is~~ not inflection points as concavity doesn't change.

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3. You invest \$100,000 now at 3% interest. How much will you have after 5 years?

(b) You invest \$200,000 now at a fixed interest rate. After 10 years your investment doubles. How much longer must you wait till your investment triples?

(c) You invest \$ P at the beginning of the year 2000 at an interest rate of 5%. What must P be in order to be able to withdraw \$10000 at the beginning of 2010, then \$ 20000 at the beginning of 2012?

4. Suppose that Lindo cafe sells 400 half kilogram bags of Colombian coffee per week when it is priced at \$10 per 500 gms. For every \$1 per bag increase in price, it sells 10 fewer bags of coffee. Recall that the price elasticity of demand is given by $e(p) = \frac{p}{q}\frac{dq}{dp}$.

(a) Find the demand equation for Lindo's Colombian coffee. Use $p$ for price and $q$ for the demand.

(b) Compute $e(p)$ for this demand function.

(c) If the price is 12 and increases by 4%, what is the percentage change in demand? You may leave your answer in the simplest calculator-ready form you can find.

(d) Will the Lindo cafeś revenue increase or decrease as a result of the price change in part (c)? Explain your answer.

Ans 4) a) $A = Pe^{rt} = 100000\, e^{0.03(5)}$

(b) $200000 = e^{r \cdot 10} = 400,000$

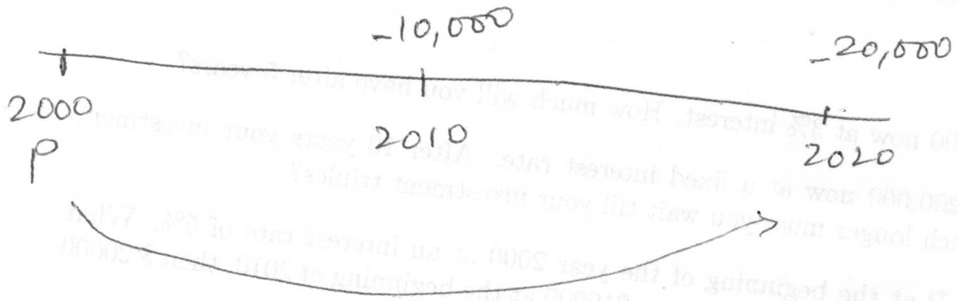
$\Rightarrow 10r = \ln 2 \Rightarrow r = \frac{\ln 2}{10}$

$200000\, e^{r(10+t)} = 600000$

$\Rightarrow r(10+t) = \ln 3 \Rightarrow t = \frac{\ln 3}{r} - 10$

$\Rightarrow t = \frac{10 \ln 3}{\ln 2} - 10$ years.

(c) 8




$$P e^{0.05(20)} - 10000\, e^{0.05(10)} - 20{,}000 = 0$$

$$\Rightarrow P = \frac{10000\, e^{0.5} + 20{,}000}{e^{0.1}}$$

Pb 4)

The Problem (4) was ~~proved~~ solved in class.