The University of British Columbia

Midterm Examination - October 4, 2012

MATH 104/184

SOLUTIONS

Closed book examination

Last Name ____________________ **First** ____________________

Student Number ____________________ **Section Number** ____________________

Signature ____________________

Special Instructions:

No memory aids, Calculators, or electronic devices of any kind are allowed. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1	10	10
2	9	9
3	7	7
4	8	8
5	8	8
6	10	10
Total		52

[10] **1.**
Find the following limits.

a) $\lim_{x\to 2} \frac{x^2-3x+2}{x^2-4}$

b) $\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}$

a) $= \lim_{x\to 2} \frac{\cancel{(x-2)}(x-1)}{\cancel{(x-2)}(x+2)} = \frac{2-1}{2+2} = \frac{1}{4}$

b) $= \lim_{x\to 0} \frac{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}{x(\sqrt{x+1}+1)} = \lim_{x\to 0} \frac{\cancel{x}+\cancel{1}-\cancel{1}}{\cancel{x}(\sqrt{x+1}+1)} = \frac{1}{\sqrt{1+0}+1} = \frac{1}{2}$

Compute the derivatives of the following functions. DO NOT SIMPLIFY.

c) $f(x) = \frac{x^2+12x+e^3}{x+e^x}$

d) $g(t) = e^{3t}(t^2+x^2)$

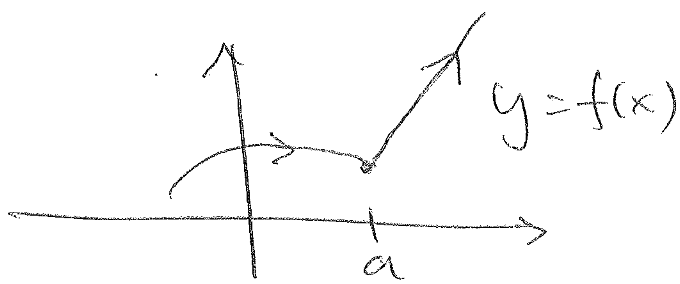
c) $f'(x) = \frac{(x+e^x)(2x+12)-(x^2+12x+e^3)(1+e^x)}{(x+e^x)^2}$

d) $g'(t) = 3e^{3t}(t^2+x^2)+e^{3t}(2t)$

[9] **2**. For each question below, either explain why the statement is true or show the statement is false by providing a counter example if appropriate.

(a) If $f(x)$ is continuous at $x = a$, then it must also be differentiable at $x = a$.

FALSE



(b) $e^x(x-1) = 1$ has a solution x in the interval $(0, 5)$.

TRUE let $f(x) = e^x(x-1)$.

1) f is continuous on $(-\infty, \infty)$

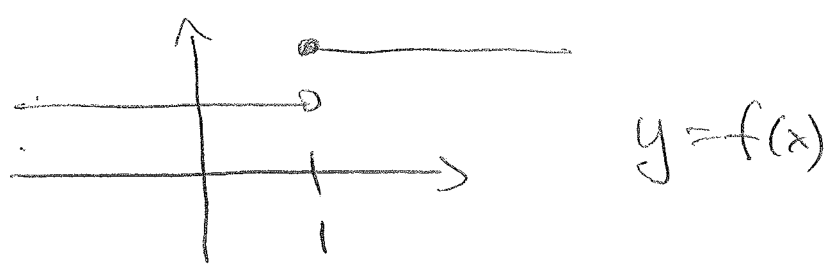
2) $f(0) = e^0(0-1) = -1 < 0$

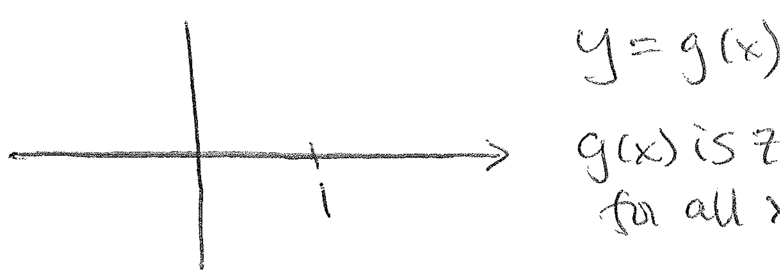
$f(5) = e^5(5-1) = e^5 \cdot 4 > 0$

3) $\therefore$ $f(c) = 0$ for some c in $(0,5)$ by I.V.T.

(c) If $f(x)g(x)$ is continuous at $x = 1$, then both $f(x)$ and $g(x)$ must also be continuous at $x = 1$.

FALSE





g(x) is zero for all x.

[7] **3.**

a) (2 marks) Carefully state the definition of the derivative of a function $f(x)$ at a point $x = a$.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$ provided the limit exists

b) (5 marks) Use the definition of the derivative from part (a) to compute $f'(1)$ for $f(x) = \dfrac{13}{x+7}$. NO CREDIT will be given for any other method.

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{13/(1+h+7) - 13/8}{h}$$

$$= \lim_{h \to 0} \frac{13 \cdot 8 - 13(8+h)}{h \cdot 8 \cdot (8+h)}$$

$$= \lim_{h \to 0} \frac{-13\cancel{h}}{\cancel{h} \cdot 8 \cdot (8+h)}$$

$$= \frac{-13}{8 \cdot 8}$$

$$= \frac{-13}{64}$$

[8] 4.
Find the values of the parameters a and b such that the function

$$f(x) = \begin{cases} (2x+a)^3, & \text{if } x \le 0, \\ 5bx + 8, & \text{if } 0 < x \le 1, \\ x^2 + 12, & \text{if } x > 1, \end{cases}$$

is continuous at all the points in its domain. Is f differentiable at all points in its domain with these values of a and b?

$$\begin{cases} \lim_{x \to 0^+} f(x) = 8 \\ \lim_{x \to 0^-} f(x) = a^3 = f(0) \end{cases}$$

$\therefore \lim_{x \to 0} f(x) = f(0)$ when $a^3 = 8$

$a = 2$

$$\begin{cases} \lim_{x \to 1^+} f(x) = 13 \\ \lim_{x \to 1^-} f(x) = 5b + 8 = f(1) \end{cases}$$

$\therefore \lim_{x \to 1} f(x) = f(1)$ when $5b + 8 = 13$

$b = 1$

$\therefore$ $f(x)$ cont. at all pts in domain when $a=2$, $b=1$.

$$\begin{cases} \lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} 5 = 5 \\ \lim_{x \to 0^-} f'(x) = \lim_{x \to 0^-} 3(2x+2)^2 \cdot 2 = 24 \neq 5 \end{cases}$$

$\therefore f'(0)$ DNE

$$\begin{cases} \lim_{x \to 1^+} f'(x) = \lim_{x \to 1^+} 2x = 2 \\ \lim_{x \to 1^-} f'(x) = \lim_{x \to 1^-} 5 = 5 \neq 2 \end{cases}$$

$\therefore f'(1)$ DNE

[8] 5. Find the equation of the tangent line to the curve $y = f(x) = \frac{1}{\sqrt[3]{x^2}}$ that is parallel to the line $y - 2x = \pi$.

$y = 2x + \pi$

$f(x) = x^{-\frac{2}{3}}$

$f'(x) = -\frac{2}{3}x^{-\frac{5}{3}}$

$f'(x) = 2 \iff -\frac{2}{3}x^{-5/3} = 2$

$\iff x^{-5/3} = -3$

$\iff x = -3^{-3/5}$

$\therefore y = (-3^{-3/5})^{-2/3}$

$= 3^{2/5}$

$\therefore$ tangent line in question has slope $= 2$

and contains point $(-3^{-3/5}, 3^{2/5})$

$\therefore$ equation of line is

$$\frac{(y - 3^{2/5})}{(x + 3^{-3/5})} = 2$$

$$y - 3^{2/5} = 2(x + 3^{-3/5})$$

[10] **6**. When EZ Electronics Company sells surge protectors at \$50 a piece, they produce and sell 3000 of them per month. For every \$1 increase in price, the number of surge protectors they sell decreases by 15. Assume that the fixed production costs are \$50,000 and the variable costs are \$30 per surge protector.

a) Find the linear demand function $q = D(p)$, where p is a price of a unit and q is the number of surge protectors made and sold. [Hint: The point $(p, q) = (50, 3000)$ must lie on this line.]

$$\frac{q-3000}{p-50} = -15$$

$$\therefore\ q - 3000 = -15(p-50),\quad q = -15p + 3750$$

also $p = -\frac{1}{15}q + 250$

b) Find the cost function $C(q)$ as a function of q, and then express it as a function of p.

$$C(q) = 50\,000 + 30q$$

$$C(p) = 50\,000 + 30(-15p + 3750)$$
$$= -450p + 162\,500$$

c) Find the revenue function $R(q)$ as a function of q, and then express it as a function of p.

$$R(q) = p(q)\cdot q = -\frac{1}{15}q^2 + 250q$$

$$R(p) = p \cdot q(p) = -15p^2 + 3750p$$

d) Find the marginal profit, $MP(p)$, with respect to p.

$$P'(p) = R'(p) - C'(p)$$
$$= -30p + 3750 + 450$$
$$= -30p + 4200$$

e) Find the price p at the *break-even points*. You may leave your answer unsimplified.

$$R(p) = C(p)$$
$$\Rightarrow -15p^2 + 3750p = -450p + 162500$$
$$\Rightarrow 15p^2 - 4200p + 162500 = 0$$
$$\Rightarrow p = \frac{4200 \pm \sqrt{4200^2 - 4 \cdot 15 \cdot 162500}}{2 \cdot 15}$$

f) If EZ Electronics Company is operating at the higher break-even point, should it increase or decrease the price of its surge protectors to increase its profits? Explain your answer.

higher break even pt, P_2, from part e)

will satisfy

$$p_2 > \frac{4200}{2 \cdot 15} = \frac{4200}{30}$$

$\therefore$ $P'(p_2) < 0$ by part d)

$\therefore$ should decrease price from P_2 in order to increase profit.