Homework 1 - due January 16th

1. It is 2030 and you are a professor at the Institute for Writing And Research Mathematics (WARM). After class at WARM one day, one of your students asks you to look over their “proof” for the problem

You have a white box with 60 white balls and a black box with 60 black balls. You take 20 balls from the white box, put them in the black box, mix the balls around and then randomly take out 20 balls and put them in the white box. In the end, which is larger: the number of black balls in the white box, or the number of white balls in the black box?

Proof. The numbers are the same because however many black balls move to the white box, that’s how many white balls move to the black box. For example, say the batch of 20 balls we move back to the white box has 11 black balls and 9 white balls, then 11 white balls stayed in the black box, by induction.

(a) In which of the following ways is this not a proof? Explain your answer.
   • It is too vague to be a rigorous proof.
   • It uses irrelevant concepts.
   • It restates the claim we are trying to prove using it as an explanation.
   • It is an example.

(b) Prove that for every positive integer $n$,

$$1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$  

Homework 2 - due January 23rd

2. One day in the coffee room, one of your fellow professors at WARM comes up to you and says “All positive integers are even! I have a proof!”

What is wrong with the following proof that they give you?

Proof. We are going to do a strong induction on the positive integers.
Assume that every positive integer up to and including $n$ is even. Then we want to show that $n + 1$ is even. By the induction hypothesis we know that $n - 1$ is even. Therefore $n - 1 = 2m$ for some integer $m > 0$. Hence $n + 1 = n - 1 + 2 = 2m + 2 = 2(m + 1)$ is even, and the result follows by induction.