Homework 1 - due January 11th

1. It is 2028 and you are a professor at the Institute for Writing And Research Mathematics (WARM). After class at WARM one day, one of your students asks you to look over their "proof" for the problem

For all $n \geq 1$, $n^3 - n$ is divisible by 6.

Proof. We can see that $5^3 - 5 = 120$ and $8^3 - 8 = 504$ are divisible by 6. Assume $k^3 - k$ is divisible by 6, then so is $(k+1)^3 - (k+1) = k^3 + 3k^2 + 2k$. We know the result $(k+1)^3 - (k+1)$ is divisible by 6. Hence by induction, for all $n \geq 1$, $n^3 - n$ is divisible by 6. □

(a) In which of the following ways is this not a proof? Explain your answer.

- It is too vague to be a rigorous proof.
- It uses irrelevant concepts.
- It restates the claim we are tying to prove using it as an explanation.
- It is an example.

(b) Three tablespoons of milk from a glass of milk are poured into a glass of tea and then thoroughly mixed. Then three tablespoons of this mixture are poured back into the glass of milk. Which is greater now: the percentage of milk in the tea, or the percentage of tea in the milk? Prove your answer.

Homework 2 - due January 18th

2. (a) One day in the coffee room, one of your fellow professors at WARM comes up to you and says "All natural numbers are even! I have a proof!"

What is wrong with the following proof that they give you?

Proof. We are going to do a strong induction on the the natural numbers.

Assume that every natural number up to and including n is even. Then we want to show that $n+1$ is even. By induction we know that $n - 1 \geq 1$ is even. We also know that $n - 1 = 0$ is even. Therefore $n - 1 = 2m$ for some integer $m \geq 0$. Hence $n+1 = n-1+2 = 2m+2 = 2(m+1)$ is even, and the result follows by induction. □

(b) Prove that for every positive integer $n \geq 5$, we have that $2^n > n^2$.

Homework 3 - due January 25th

3. For LaTeX practice, transcribe all of your research journals so far into one long document in LaTeX. Do the following.

- Each weekly entry should start a new page. It should say which journal entry it is as a title. Your name and student number should also be on this page.
- There should be three sections within each entry: What I did; Why I did it; What obstacles I encountered and my research plan for the following week.
- ***From now on:*** Please latex up all submissions including your homework and your journal entry. In all cases submit the PDF created. Thank you.

Homework 4 - due February 1st

4. Interview one of your mathematics professors (past or present, but not me as we have talked about this together already) for about 15 minutes. Ask them the following.

(a) Where do they get ideas for a new problem to work on, and how do they start working on it?

(b) What do they do if they get stuck?

Write up a summary in LaTeX of who you interviewed, and what did they say, to hand in.