Homework 1 - due January 11th

1. It is 2028 and you are a professor at the Institute for Writing And Research Mathematics (WARM). After class at WARM one day, one of your students asks you to look over their “proof” for the problem

For all \( n \geq 1 \), \( n^3 - n \) is divisible by 6.

**Proof.** We can see that \( 5^3 - 5 = 120 \) and \( 8^3 - 8 = 504 \) are divisible by 6. Assume \( k^3 - k \) is divisible by 6, then so is \( (k + 1)^3 - (k + 1) = k^3 + 3k^2 + 2k \). We know the result \( (k + 1)^3 - (k + 1) \) is divisible by 6. Hence by induction, for all \( n \geq 1 \), \( n^3 - n \) is divisible by 6.

(a) In which of the following ways is this not a proof? Explain your answer.

- It is too vague to be a rigorous proof.
- It uses irrelevant concepts.
- It restates the claim we are trying to prove using it as an explanation.
- It is an example.

(b) Three tablespoons of milk from a glass of milk are poured into a glass of tea and then thoroughly mixed. Then three tablespoons of this mixture are poured back into the glass of milk. Which is greater now: the percentage of milk in the tea, or the percentage of tea in the milk? Prove your answer.

Homework 2 - due January 18th

2. (a) One day in the coffee room, one of your fellow professors at WARM comes up to you and says “All natural numbers are even! I have a proof!”

*What is wrong with the following proof that they give you?*

**Proof.** We are going to do a strong induction on the natural numbers.

Assume that every natural number up to and including \( n \) is even. Then we want to show that \( n + 1 \) is even. By induction we know that \( n - 1 \geq 1 \) is even. We also know that \( n - 1 = 0 \) is even. Therefore \( n - 1 = 2m \) for some integer \( m \geq 0 \). Hence \( n + 1 = n - 1 + 2 = 2m + 2 = 2(m + 1) \) is even, and the result follows by induction.

(b) Prove that for every positive integer \( n \geq 5 \), we have that \( 2^n > n^2 \).