Homework 1 - due January 12th

1. It is 2027 and you are a professor at the Institute for Writing And Research Mathematics (WARM). After class at WARM one day, one of your students asks you to look over their “proof” for the problem

You have a white box with 60 white balls and a black box with 60 black balls. You take 20 balls from the white box, put them in the black box, mix the balls around and then randomly take out 20 balls and put them in the white box. In the end, which is larger: the number of black balls in the white box, or the number of white balls in the black box?

Proof. The numbers are the same because however many black balls move to the white box, that’s how many white balls move to the black box. For example, say the batch of 20 balls we move back to the white box has 11 black balls and 9 white balls, then 11 white balls stayed in the black box, by induction.

(a) In which of the following ways is this not a proof? Explain your answer.

• It is too vague to be a rigorous proof.
• It uses irrelevant concepts.
• It restates the claim we are trying to prove using it as an explanation.
• It is an example.

(b) You have a white box with 60 white balls and a black box with 60 black balls. You take 20 balls from the white box, put them in the black box, mix the balls around and then randomly take out 20 balls and put them in the white box. Then you do this again. In the end, which is larger: the number of black balls in the white box, or the number of white balls in the black box? Prove your answer.

Homework 2 - due January 19th

2. (a) One day in the coffee room, one of your fellow professors at WARM comes up to you and says “All babies have the same colour eyes! I have a proof!”

What is wrong with the following proof that they give you?

Proof. We are going to do a strong induction on the number of babies.

Base case: $n = 1$ is clearly true.

Induction step: Assume that every set of up to $n$ babies has the same colour eyes. Consider a set of $n + 1$ babies sitting in a straight line. By induction the $n$ leftmost babies have the same colour eyes. Similarly by induction the $n$ rightmost babies have the same colour eyes. Then all $n + 1$ babies have the same colour eyes, as the rightmost baby and the leftmost baby have the same colour eyes as all the babies in between. Hence, by induction, every set of $n$ babies has the same colour eyes for all $n \geq 1$. Since the total number of babies is some number $N \geq 1$ the result follows.
(b) Prove that for every positive integer $n$,

$$1^3 + 2^3 + \cdots + n^3 = \left( \frac{n(n + 1)}{2} \right)^2.$$

**Homework 3 - due January 26th**

3. For \LaTeX{} practice, transcribe all of your research journals so far into one long document in \LaTeX{}.

- Each weekly entry should start a new page. It should say which journal entry it is as a title. Your name and student number should also be on this page.
- There should be three sections within each entry: What I did; Why I did it; What obstacles I encountered and my research plan for the following week.
- *From now on* Please latex up all submissions including your homework and your journal entry. In all cases submit the PDF created. Thank you.

**Homework 4 - due February 2nd**

4. Interview one of your mathematics professors (past or present, but not me as we have talked about this together already) for about 15 minutes. Ask them the following.

(a) Where do they get ideas for a new problem to work on, and how do they start working on it?

(b) What do they do if they get stuck?

Write up a summary in \LaTeX{} of who you interviewed, and what did they say, to hand in.

**Homework 5 - due February 9th**

5. One of your colleagues at WARM tells you about an old theorem they read about:

*If $n + 1$ or more objects are put into $n$ boxes, then at least one box contains more than one object.*

Use this to prove the following.

- From the integers 1, 4, 7, 10, 13, \ldots, $3k + 1$, \ldots, 100 there are 20 different ones chosen at random. Prove that among those chosen there exists two that add up to 101.

**Homework 6 - due March 2nd**

6. Download a copy of “Polygon Dissections and Standard Young Tableaux” and

- rewrite the abstract so it has the structure of an abstract as we discussed in class;
- state 3 ways in which the local structure of this paper could be improved.
Homework 7 - due March 9th

7. *Interesting induction:* Prove that every positive integer $n$ can be written as a sum of 4’s and 5’s for all $n \geq 12$.

*Example:* $13 = 4 + 4 + 5$. 