1.

2. Build two bridges given by the dotted lines, and take the route given by taking the bridges in the order below.

3. Observe that on an odd board there will always be one more square of one colour (say black) than the other, and a knight will always move from a square to a square of the other colour. Hence one can never find a knight’s tour on an odd board since to get to all the squares once you would need to move

\[ BWBWBW \ldots WB \]

but there is no way to return from a black square to a black square without visiting an already visited white square.
4. From 1 we have 3 choices to go to $x$. From here we have 2 choices, then from here we have 2 choices and then our path is determined from there, so the total number is $3 \cdot 2 \cdot 2 = 12$.

5. Each vertex has 3 “lives”. After no moves the total number of lives left is $3n$. After one move the total number of lives left is $3(n + 1) - 4$. After 2 moves it is $3(n + 2) - 8$ and after $p$ moves it is $3(n + p) - 4p$. The game must end when there are no lives left, i.e. $3(n + p) - 4p = 0$ or $3n = p$. However the last vertex added can’t have had all its lives used up, so the number of moves must be less than $3n$ i.e. at most $3n - 1$.

6. No.