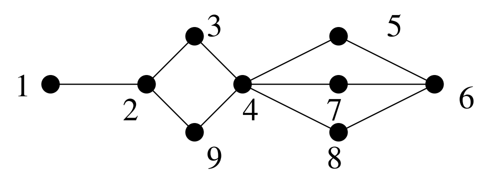
**1.** (a) The number of pairs $(T, e)$ is the number of spanning trees of $K_n$ multiplied by the number of edges in a spanning tree. By Cayley's formula and Q51 this is $n^{n-2}(n-1)$.

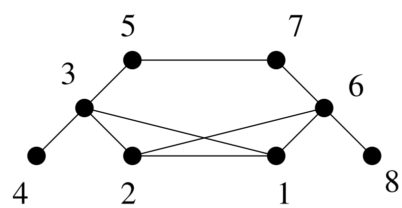
(b) By Proposition 1, the number of edges in $K_n$ is $\frac{1}{2}n(n-1)$. So the number of spanning trees containing a specific edge is the total number of spanning trees of $K_n$ with a focus on a specific edge, divided by the number of edges in $K_n$ (by symmetry of $K_n$), that is,

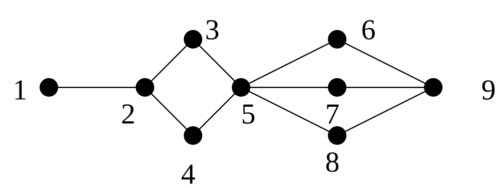
$$\frac{n^{n-2}(n-1)}{\frac{1}{2}n(n-1)} = 2n^{n-3}.$$

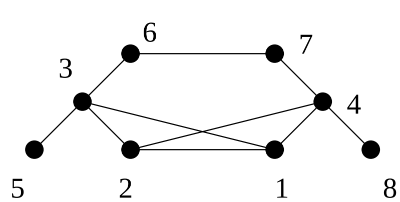
(c) Hence the number of spanning trees not containing a specific edge is the total number of spanning trees of $K_n$ minus the number of spanning trees containing the specific edge or

$$n^{n-2} - 2n^{n-3} = (n-2)n^{n-3}.$$

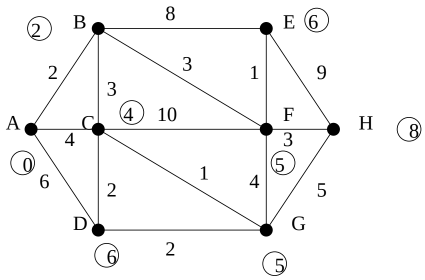


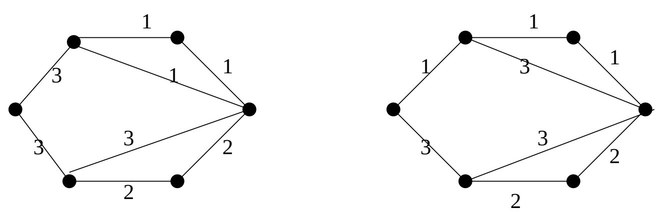

**2.**




**3.**

**4.** If the total weight on every cycle is even then in particular the total weight on every triangle is even.
Conversely, let us do an induction on the length of the cycle, $n$.
Base case: $n = 3$ is fine since the weight of every triangle is even.

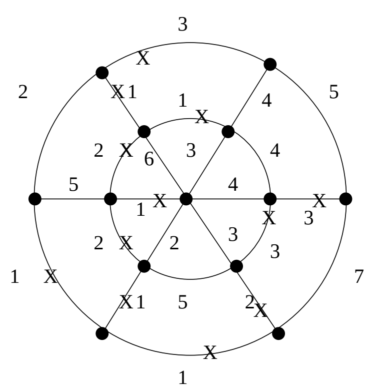
Induction step: Assume true for $n = k$. For $n = k + 1$ make the cycle by removing an edge $e$ from a cycle of length $k$ and inserting the other 2 sides of the triangle $f, g$ (since we have a complete graph). If $w(e)$ was odd then $w(f) + w(g)$ is odd since $w(e) + w(f) + w(g)$ is even, and if $w(e)$ was even then $w(f) + w(g)$ is even since $w(e) + w(f) + w(g)$ is even. In both cases the weight of the cycle is even and we are done.



not unique

unique

**5.**



Pick the X edges.

**6.**