The remainder of this page has been left blank for your workings.
1. (4 points) Determine whether each of the following statements is TRUE or FALSE. [No explanation is needed.]

(a) If the series $\sum_{k=0}^{\infty} a_k z^k$ converges at $z = 2i$, then its radius of convergence is at least 2.

Answer:

(b) The Taylor series of $e^z$ about $z = 0$ converges uniformly over the unit disk.

Answer:

(c) The function $\log z$ has a Laurent expansion about $z = 0$ that is convergent in the punctured unit disk $0 < |z| < 1$.

Answer:

(d) The function $\frac{1 - \cos z}{z^3}$ has a pole of order 3 at $z = 0$.

Answer:
2. (3 points) Find a Laurent expansion of \( g(z) := \frac{2(z + 2)}{z^2 - 1} \) about \( z = -2 \) that is convergent at \( z = 0 \).
3. (3 points) Show that the series \( \sum_{k=0}^{\infty} \frac{z^k}{1 + z^{2k}} \) converges if and only if \( z \) is not on the unit circle.