1. (4 points) Determine whether each of the following statements is TRUE or FALSE. [No explanation is needed.]

(a) $\log(z_1z_2) = \log z_1 + \log z_2$ for every $z_1, z_2 \in \mathbb{C} \setminus \{0\}$.

Answer:

(b) $e^{\log z} = z$ for every $z \in \mathbb{C} \setminus \{0\}$, and $\log e^z = z$ for every $z \in \mathbb{C}$.

Answer:

(c) The coefficient of $1/z$ in the partial fraction decomposition of $R(z) := \frac{z - 1}{z^2(z + 1)}$ is 2.

Answer:

(d) If $z \neq 0$ and $\alpha \in \mathbb{R}$, then all the values of $z^\alpha$ are on a circle centered at the origin, and all the values of $z^{i\alpha}$ are on a ray emitting from the origin.

Answer:
2. (2 points) Argue that \( u(x, y) := (\ln |z|)^2 - (\text{Arg } z)^2 \) (where \( z = x + iy \)) is harmonic in the domain \( D^* := \mathbb{C} \setminus \{ x + iy : x \leq 0 \text{ and } y = 0 \} \).

[HINT: Recall that the real and imaginary parts of an analytic function are harmonic.]
3. (4 points) Determine the domain of analyticity for \( f(z) := \log(e^{iz} + 1) \). Sketch the set of points at which \( f \) is not analytic and compute \( f'(0) \).