MATH 300.202
Assignment 3
Due: Wednesday, January 25

I. Section 1.6: 17, 18, 20

II. Section 2.1: 4, 6, 10, 12

III. Section 2.2: 7(c, e, f), 12, 14

IV. Find each of the following limits or specify if the limit does not exist.

(a) \( \lim_{z \to i} z^3 + 3iz + 1 - 2i \)
(b) \( \lim_{z \to -i} \frac{z^4 - 1}{z + 1} \)
(c) \( \lim_{z \to 1+i} \frac{z^2 + z - 1 - 3i}{z^2 - 2z + 2} \)
(d) \( \lim_{z \to 0} \frac{z^2}{z \overline{z}} \)

V. Determine where the following functions are continuous.

(a) \( f(z) := \text{Arg}(z) \)
(b) \( f(z) := z + \overline{z} \)
(c) \( f(z) := \frac{1}{|z|^2 - 1} \)
(d) \( f(z) := \begin{cases} \frac{z^3 + 1}{z + 1} & \text{if } z \neq 1, \\ 1 & \text{otherwise.} \end{cases} \)

VI. Show that the geometric inversion mapping \( f(z) := 1/z \) (a.k.a., reflection in the unit circle) maps every line not passing through the origin onto a circle passing through the origin excluding the origin itself. (For simplicity, you may assume that the line does not intersect the unit circle.) [HINT: Consider the images of two points \( z_0 \) and \( z \) on the line \( \ell \), chosen such that the vector \( z_0 \) is perpendicular to the line \( \ell \) and \( z \) is distinct from \( z \). Look for similar triangles and determine the locus of \( f(z) \) as \( z \) varies over \( \ell \).]