Question 1. Given a complete weighted graph on $2n$ vertices, $K_{2n}$. The weights of the edges are positive numbers satisfying the triangle inequality. We choose $2m$ vertices ($m \leq n$) and among them we select the minimum weight perfect matching. (The weight of the perfect matching - denoted by $W_{2m}$ - is the sum of the weights of the $m$ selected edges)

(1) Prove or give a counterexample that the weight of any perfect matching in $K_{2n}$ is at least $W_{2m}$:

(2) Is it true that the weight of the Minimum Perfect Matching in $K_{2n}$ is at most the weight of the Minimum Spanning Tree?

Question 2.* Given a complete weighted graph on $n$ vertices, $K_n$: The weights of the edges are positive numbers satisfying a strong triangle inequality; $a + b \geq 1.1c$ instead of the usual $a + b \geq c$ for any triangle with edge weights $a, b$ and $c$. Can you improve Christofides' 1.5-approximation for the Traveling Salesman Problem in this graph?

Question 3. Give an unsatisfiable 3SAT example with 3 variables and at most 10 clauses. Each clause must contain exactly three literals. Prove that the formula is unsatisfiable.

Question 4. Is there an unsatisfiable 3SAT example where no clauses contain the same

(1) three variables?

(2) two variables? (Any two clauses contain at most one common variable)

Justify your answer.

Due date: Sept. 21 in class