Question 1. Let \( S \subset 2^H \) a collection of subset of \( H \) so that if \( A, B, C, D \in S \) are distinct sets then \( A \cup B \not\subset C \cup D \). Give upper and lower bounds on the size of \( S \) in terms of \(|H|\) if we know that \(|S|\) is as large as possible under the given condition.

Question 2. Given a simple graph, \( G_n \), with \( e \) edges.
   (1) Give a lower bound on the number of \( K_{2,3} \) subgraphs in terms of \( n \) and \( e \).
   (2) What is the expected number of \( K_{2,3} \) subgraphs if \( G_n \) is a random graph? Every edge was selected independently at random with probability
   \[
   p = \frac{e}{\binom{n}{2}}.
   \]

Question 3. Draw your student tree! i.e. write down your student number and consider it as a Pruefer code of a labelled tree. If you don’t have a proper student number then you can work with the code 02309111.

Question 4. Prove the identity that if \( k = n \) then
\[
\sum_{i=0}^{n} (-1)^i \binom{n}{i} i^k = (-1)^n n!
\]
What happens if \( k \neq n \)?

Due date: Oct 5, in class.