# MATH 503 HW 2

**Question 1.** Let $S \subset 2^H$ a collection of subset of $H$ so that if $A, B, C, D \in S$ are distinct sets then $A \cup B \not\subset C \cup D$. Give upper and lower bounds on the size of $S$ in terms of $|H|$ if we know that $|S|$ is as large as possible under the given condition.

**Question 2.** Given a simple graph, $G_n$, with $e$ edges.

(1) Give a lower bound on the number of $K_{2,3}$ subgraphs in terms of $n$ and $e$.
(2) What is the expected number of $K_{2,3}$ subgraphs if $G_n$ is a random graph? Every edge was selected independently at random with probability

$$p = \frac{e}{\binom{n}{2}}.$$

**Question 3.** Draw your student tree! i.e. write down your student number and consider it as a Pruefer code of a labelled tree. If you don't have a proper student number then you can work with the code 02309111.

**Question 4.** Prove the identity that if $k = n$ then

$$\sum_{i=0}^{n} (-1)^i \binom{n}{i} i^k = (-1)^n n!$$

What happens if $k \neq n$?

Due date: Oct 5, in class.