MATH 503 HW 5.

There are four questions to answer.

Question 1. For each equation below prove or disprove that for any $k \geq 1$ there is a threshold, $n_0 = n_0(k)$ such that if $n \geq n_0$ then any k-colouring of the first n integers contains three numbers $x, y, z \in [n]$ from the same colour class giving solution to the equation

(1) $2x^2 + y = z^2$
(2) $x + y = 3z$
(3) $xy = z^2$
(4) In this question we colour all real numbers with k colours. Is it true that for any colouring there are three numbers $x, y, z \in \mathbb{R}$ from the same colour class giving solution to the equation $\frac{z-y}{y-x} = \frac{z-x}{z-y}$?

Question 2. The Hales-Jewett theorem asserts that if $d \geq HJ(k, \ell)$ then any k-colouring of C_ℓ^d contains a monochromatic combinatorial line. Give upper and lower bounds on $HJ(k, 2)$.

Question 3. Show that for any integer $k \geq 2$ there is a 4-uniform hypergraph on n vertices such that

- any two edges have at most one common vertex
- the maximum degree is at most $2n^{1/3}$,
- the chromatic number of the graph is at least k.

Question 4. True or false? If a connected graph is large enough and has max degree 100, then no matter how do we colour the vertices with k colours there will be three distinct vertices v_1, v_2, v_3 from the same colour class so that $dist(v_1, v_2) = dist(v_2, v_3)$ where the distance between two vertices is the length of the shortest path connecting them.

Due date: Nov 22, in class.