There are four questions to answer.

**Question 1.** For each equation below prove or disprove that for any \( k \geq 1 \) there is a threshold, \( n_0 = n_0(k) \) such that if \( n \geq n_0 \) then any \( k \)-colouring of the first \( n \) integers contains three numbers \( x, y, z \in [n] \) from the same colour class giving solution to the equation

\[
\begin{align*}
(1) \quad 2x^2 + y &= z^2 \\
(2) \quad x + y &= 3z \\
(3) \quad xy &= z^2 \\
(4) \text{In this question we colour all real numbers with } k \text{ colours. Is it true that for any colouring there are three numbers } x, y, z \in \mathbb{R} \text{ from the same colour class giving solution to the equation } \frac{z-y}{y-x} = \frac{z-x}{z-y} ?
\end{align*}
\]

**Question 2.** The Hales-Jewett theorem asserts that if \( d \geq HJ(k, \ell) \) then any \( k \)-colouring of \( C_d^\ell \) contains a monochromatic combinatorial line. Give upper and lower bounds on \( HJ(k, 2) \).

**Question 3.** Show that for any integer \( k \geq 2 \) there is a 4-uniform hypergraph on \( n \) vertices such that

- any two edges have at most one common vertex
- the maximum degree is at most \( 2n^{1/3} \)
- the chromatic number of the graph is at least \( k \).

**Question 4.** True or false? If a connected graph is large enough and has max degree 100, then no matter how do we colour the vertices with \( k \) colours there will be three distinct vertices \( v_1, v_2, v_3 \) from the same colour class so that \( \text{dist}(v_1, v_2) = \text{dist}(v_2, v_3) \) where the distance between two vertices is the length of the shortest path connecting them.

Due date: Nov 22, in class.