MATH 503 HW M

Question 1. Prove the following binomial identity;

$$\sum_{k=0}^{n} \binom{n+k}{n} = \binom{2n+1}{n}$$

Question 2. What is the expected number of leaves in a random labeled tree on n vertices? Random labeled trees are selected from a uniform distribution. Thus, given n, each of the n^{n-2} trees is equally likely to be chosen.

Question 3. We put 2015 billiard balls into a (three-dimensional) bin. What is the smallest number of colours needed to guarantee a colouring of the balls so that no touching balls receive the same colour? All balls are of equal size. (We assign the colours after placing them into the bin.)

Question 4. Prove that for every integer $k \geq 3$ there is a constant $c_k > 0$ such that any n-element planar point set contains $m \geq c_k n$ points $p_1, \ldots, p_m$ such that any $k+1$ consecutive points $p_i, p_{i+1}, \ldots, p_{i+k}$ $(0 < i \leq m-k)$ are in convex position.

Question 5. Give upper and lower bounds on the Ramsey number $R = R(\underbrace{4, 4, \ldots, 4}_{k})$. R is the least number such that no matter how we k colour the edges of a complete graph on R vertices, there will be four vertices so that all edges connecting them have the same colour. Give the best bound you can prove.

Question 6. Give a lower bound on the function $r(n)$, where $r(n)$ is the largest number such that no matter how we place n axis-parallel rectangles on the plane, either $r(n)$ of them will be pairwise intersecting or $r(n)$ of them will be pairwise disjoint.

Due date: Oct. 22, in class.