MATH 503

Question 1. What is the probability that a random permutation of n objects have a fixpoint? (Every permutation is selected with the same, $\frac{1}{n!}$, probability.) What is the expected number of fixpoints?

Question 2. Prove that $\binom{n}{k}$ and $\binom{2n}{2k}$ have the same parity.

Question 3. Use Szemerédi's regularity lemma to prove that for every $\varepsilon > 0$ there exists a $\delta > 0$ such that, for any graph G_n on n vertices with at most δn^3 triangles, it may be made triangle-free by removing at most εn^2 edges.

Question 4. Use the eigenvalues of the adjacency matrix to give an exact formula for the number of pentagons (C_5-s) in a graph.

Question 5. Let us suppose that the second largest eigenvalue of a d-regular bipartite graph is $2\sqrt{d}$. The two vertex sets have size $n - n$. Give upper and lower bounds on the number of quadrilaterals (C_4-s) in the graph.

Question 6.* Show that there is a constant, $c > 0$, such that the following holds; let P and Q be a set of points and a set of axis parallel rectangles in the plane, respectively. Let us suppose that every rectangle contains at least one point. Then for any natural number n, either

- there are n rectangles in Q such that no point of P belongs to more than one of them, or
- one can choose at most n^c points of P so that every element of Q contains at least one of them.

Due date: Dec. 11, 11:00 pm. Please send your solutions by email.