MATH 503 7.

Due date: Nov. 26, 2015

Problem 1. *Find the eigenvalues of the*

- *pentagon graph,* C_5,
- *the complete bipartite graph* $K_{2,3}$,
- *the path* P_5.

Problem 2. *In the following question perform all steps as required. You can use a calculator or a computer. Your task is to count the number of edges between the top four vertices and the bottom four vertices of the graph of a 3 dimensional cube, C (fig. 1). There are clearly 4 edges, but you should use linear algebra. Multiply the adjacency matrix with the indicator vectors of the two vertex sets. Don't actually multiply the matrix, use the eigenvectors instead. So, find an orthonormal basis of eigenvectors and the projections (coordinates) of the indicator vector. Then calculate the inner products of the vectors to get the desired result.*

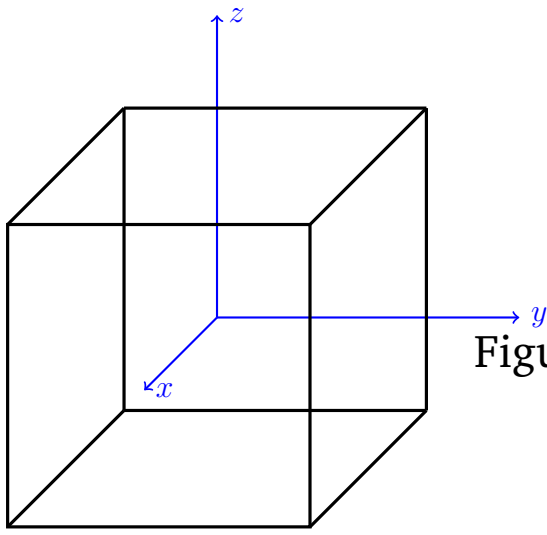
$C =$ 

Figure 1

Problem 3. *Construct a graph,* G_{29}, *on 29 vertices as follows;* $(v_i, v_j) \in E(G_{29})$ *if* $|i-j| \equiv 7 \pmod{29}$ *or* $|i-j| \equiv 11 \pmod{29}$. *Find the eigenvalues and eigenvectors of the graph. (Note that the eigenvalues are real numbers. Don't provide an exponential sum only)*