**Question 1.** Given a corner in $\mathbb{R}^3$ spanned by the points $(0,0,0), (1,0,0), (0,1,0), (0,0,1)$. Find a plane, $ax + by + cz = d$, such that the projection of the four points of the corner is a square. (Hint: write down and solve the system of linear equations representing the problem)

**Question 2.** Given an $\varepsilon$-regular bipartite graph, $G(A,B)$, with vertex sets $|A| = |B| = n$. The number of edges is $\delta \cdot n^2$. Give a lower bound on the number of quadrilaterals, $K_{2,2}$-s, in terms of $\varepsilon$ and $\delta$.

**Question 3.** Given an $\varepsilon$-regular bipartite graph, $G(A,B)$, with vertex sets $|A| = |B| = n$. The number of edges is $\delta \cdot n^2$. Give an UPPER bound on the number of quadrilaterals, $K_{2,2}$-s, in terms of $\varepsilon$ and $\delta$.

**Question 4.** For a point $c = (c_1, c_2, \ldots, c_k) \in \{1, 2, \ldots, n\}^k$ we define a jack $J(c)$ with centre $c$ as the set of points that differ from $c$ in at most one coordinate. For $i, 1 < i < k$, and fixed $c_1, c_2, \ldots, c_{i-1}, c_{i+1}, \ldots, c_k \in 1, 2, \ldots, n$, we also define a line as a set of $n$ points of the form $\{(c_1, c_2, \ldots, c_{i-1}, x, c_{i+1}, \ldots, c_k) : 1 < x < n\}$.

Let $LS(n, k)$ be the maximum cardinality of a system $J$ of jacks for which no two distinct jacks share a common line, and every $k$ distinct jacks from $J$ have an empty intersection. Prove that $LS(n, k)/n^{k-1}$ tends to 0 as $n \to \infty$.

Due date: Nov 12, in class.