MATH 503 HW 6

Question 1. Given a corner in $\mathbb{R}^3$ spanned by the points $(0,0,0),(1,0,0),(0,1,0),(0,0,1)$. Find a plane, $ax+by+cz=d$, such that the projection of the four points of the corner is a square. (Hint: write down and solve the system of linear equations representing the problem)

Question 2. Given an ε-regular bipartite graph, $G(A,B)$, with vertex sets $|A|=|B|=n$. The number of edges is $\delta \cdot n^2$. Give a lower bound on the number of quadrilaterals, $K_{2,2}$-s, in terms of ε and δ.

Question 3. Given an ε-regular bipartite graph, $G(A,B)$, with vertex sets $|A|=|B|=n$. The number of edges is $\delta \cdot n^2$. Give an UPPER bound on the number of quadrilaterals, $K_{2,2}$-s, in terms of ε and δ.

Question 4. For a point $c=(c_1,c_2,...,c_k)\in\{1,2,...,n\}^k$ we define a jack $J(c)$ with centre c as the set of points that differ from c in at most one coordinate. For $i, 1<i<k$, and fixed $c_1,c_2,...,c_{i-1},c_{i+1},...,c_k \in 1,2,...,n$, we also define a line as a set of n points of the form $\{(c_1,c_2,...,c_{i-1},x,c_{i+1},...,c_k): 1<x<n\}$.

Let $LS(n,k)$ be the maximum cardinality of a system J of jacks for which no two distinct jacks share a common line, and every k distinct jacks from J have an empty intersection. Prove that $LS(n,k)/n^{k-1}$ tends to 0 as $n\to\infty$.

Due date: Nov 12, in class.