MATH 503 HW 5

Question 1. Prove the following bound using the Lovász Local Lemma: The van der Waerden number $W(k)$ for two colors satisfies $W(k) \geq 2^k/8k$. (If you get a weaker bound only then explain that)

Question 2. Give an upper and a lower bound on the number of convex quadrilaterals determined by n points in the plane. So, find two functions f and g such that any n-element pointset contains at least $f(n)$ convex four-gons and there is a pointset for every n where the number of convex four-gons is at most $g(n)$. (We can suppose that no three points are on a line)

Question 3. Give an upper bound on the function $r(n)$, where $r(n)$ is the largest number such that no matter how we place n axis-parallel rectangles on the plane, either $r(n)$ of them will be pairwise intersecting or $r(n)$ of them will be pairwise disjoint.

Question 4. Give a lower bound on the Ramsey number $R = R(\underbrace{r, r, \ldots, r}_{k})$. R is the least number such that no matter how we k colour the edges of a complete graph on R vertices, there will be r vertices so that all edges connecting them have the same colour. Give the best bound you can prove.

Due date: Oct. 29, in class.