MATH 503 HW 3

Question 1. Answer the following question using the Lovász Local Lemma. Given a positive integer $k \in \mathbb{Z}$. How many colours do we need to colour the integer grid, $\mathbb{Z} \times \mathbb{Z}$, such that every square of side length $r = \sqrt{k}$ has four different different colours in its vertices. The squares might not be axis parallel. (The squares are spanned by $\mathbb{Z} \times \mathbb{Z}$.)

Question 2. What is the probability that a random sequence of length $n-2$ is the Prüfer code of a path? Every entry of the sequence is chosen independently at random with equal $(1/n)$ probability from the set $\{1, \ldots, n-1, n\}$.

Question 3. In class we sketched the following bound on the diagonal Ramsey number $R(k,k)$: ... If we set n such that with the random colouring of K_n (colour the edges with red or blue independently at random with probability 1/2) the expected number of monochromatic k-cliques is at most n/k then we have $R(k,k) \geq n - n/k$. What is the bound we get from this? Use Stirling's formula to perform the exact calculations.

Question 4. A planar curve γ is x-monotone if any vertical line either does not intersect γ, or it intersects γ in a single point. It is y-monotone if any horizontal line either does not intersect γ, or it intersects γ in a single point. If both conditions hold then γ is xy-monotone. Give a lower bound on the function $g(n)$ which is defined as follows; it doesn't matter how we select n points with distinct x and y coordinates, at least $g(n)$ of them lie on an xy-monotone continuous planar curve γ.

Due date: Oct. 8, in class.