MATH 503 HW 2

Question 1. Let us colour all subsets of an n-element set by $n-1$ colours.

(1) Show that there are two sets, A and B having the same colour so that one is a subset of the other, $A \subset B$.
(2) What is the expected number of such monochromatic $A \subset B$ pairs if we colour the sets independently at random using the $n-1$ colours with equal $(1/(n-1))$ probability for each set?
(3) Answer the previous two questions if we use $k \leq n$ colours instead of $n-1$.

Question 2. What is the probability that a random sequence of length $n-2$ is the Prüfer code of a star? Every entry of the sequence is chosen independently at random with equal $(1/n)$ probability from the set $\{1, \ldots, n-1, n\}$.

Question 3.* Warning: *This is probably a hard question. Don't worry if you can't solve it.*

Let $z_1, \ldots, z_n$ be complex numbers with $|z_i| \geq 1$ for each i. Give a bound on the number of sums

$$\sum_{i=1}^{n} e^{2\pi \mathrm{i} \frac{k_i}{3}} z_i$$

lying inside a circle centered at the origin of unit radius. ($0 \leq k_i < 3$, the k_i-s are integers) With different words, how many possible sequences of third roots of unity are there where the

$$\left| \sum_{i=1}^{n} e^{2\pi \mathrm{i} \frac{k_i}{3}} z_i \right| \leq 1$$

inequality holds.

Due date: Oct. 1, in class.