MATH 503 HW 1

Where to find the solutions? Most of them are in Lovász' book. Check the solutions in the book.

Question 1. Find a formula for the number of subsets of an n element set with cardinality divisible by 3.

A similar question (with 7 instead of 3) is solved in **1.42.**(f).

Question 2.* Find a formula for the number of connected graphs with exactly one cycle on vertices labeled 1 to n.

There is a nice book where among many other interesting results you can find the answer; ASYMPTOPIA by Spencer and Florescu. If you are not that motivated then follow the link below.

http://math.umn.edu/ jblewis/4707docs/UnicyclicGraphs.pdf

Question 3. Prove that $\left\{ {n \atop n-k} \right\}$ is a polynomial in n for each fixed k.

1.6.(b).

Question 4. Prove the following identities. Use combinatorial considerations if you can.

(1)
$$\sum_{k=0}^{m} \binom{m}{k} \binom{n+k}{m} = \sum_{k=0}^{m} \binom{m}{k} \binom{n}{k} 2^k.$$

1.43.(a).

(2)
$$\sum_{k=1}^{n-1} \binom{n}{k} k^{k-1} (n-k)^{n-k-1} = 2(n-1)n^{n-2}.$$

1.44.(c).