MATH 503 HW 1

**Question 1.** Find a formula for the number of subsets of an $n$ element set with cardinality divisible by 3.

**Question 2.** Find a formula for the number of connected graphs with exactly one cycle on vertices labeled 1 to $n$.

**Question 3.** Prove that $\binom{n}{n-k}$ is a polynomial in $n$ for each fixed $k$.

**Question 4.** Prove the following identities. Use combinatorial considerations if you can.

1. \[
\sum_{k=0}^{m} \binom{m}{k} \binom{n+k}{m} = \sum_{k=0}^{m} \binom{m}{k} \binom{n}{k} 2^{k}.
\]

2. \[
\sum_{k=1}^{n-1} \binom{n}{k} k^{k-1} (n-k)^{n-k-1} = 2(n-1)n^{n-2}.
\]

Due date: Sept. 24, in class.