

MATH 503 HW 1

Question 1. Find a formula for the number of subsets of an n element set with cardinality divisible by 3.

Question 2.* Find a formula for the number of connected graphs with exactly one cycle on vertices labeled 1 to n .

Question 3. Prove that $\binom{n}{n-k}$ is a polynomial in n for each fixed k .

Question 4. Prove the following identities. Use combinatorial considerations if you can.

(1)

$$\sum_{k=0}^m \binom{m}{k} \binom{n+k}{m} = \sum_{k=0}^m \binom{m}{k} \binom{n}{k} 2^k.$$

(2)

$$\sum_{k=1}^{n-1} \binom{n}{k} k^{k-1} (n-k)^{n-k-1} = 2(n-1)n^{n-2}.$$

Due date: Sept. 24, in class.