Question 1.
**Question 2.** the first one: \( v = 8, e = 12, f = 6 \); the second one: \( v = 5, e = 9, f = 6 \)

**Question 3.** Let \( G \) be a planar graph and let \( v_\infty \) be the vertex in the dual graph \( D(G) \) representing the infinite face of \( G \). Let \( v \) be a vertex in the dual graph \( D(G) \) representing an interior face \( F \) of \( G \). Pick a point \( P_1 \) inside the face \( F \) and pick another point \( P_2 \) in the infinite face. Draw a curve on the plane connecting \( P_1 \) and \( P_2 \), without intersecting any vertex of \( G \). This curve gives a walk from \( v \) to \( v_\infty \) in \( D(G) \). Since \( v \) is arbitrary, we may conclude that every vertex of \( D(G) \) is connected to \( v_\infty \), and so the graph \( D(G) \) is connected.
Question 4.

It is clear that in $K_4$, no two vertices can share a colour.

Question 5. Notice that each crossing comes from 4 vertices. If we remove any one of these 4 vertices (and the edges adjacent to it), the crossing should be removed as well.

Suppose there is a drawing of $K_6$ with only one crossing. Then by removing any vertex corresponding to this crossing, we get a drawing of $K_5$ without any crossing. Contradiction.

Suppose there are exactly two crossings in a drawing of $K_6$. Since there are only 6 vertices in total, there are at least 2 vertices such that each correspond to both crossings. Again, by removing any of these vertices, we get a drawing of $K_5$ without any crossing. Contradiction.