MATH 309 TOPICS, QUESTIONS

Euler's formula,Convexity,Graphs, Trees,Convex Polyhedron,Planar Graphs, Drawings,Colouring.

Question What is the definition of *convex hull* ?

Question When is it true that the union of two distinct unit balls in $\mathbb{R}^3$ is a convex set ?

Question What is the chromatic number, $\chi(G_n)$, of a graph G_n?

Question Find the convex hull of the pointset $(1,-2),(3,4),(1,1),(-2,-1),(5,4),(-3,1)$.

Question Let us suppose that a simple planar graph, G_n, has only facets with even number of edges on the boundary. ($f_{2i+1} = 0$ for any $i \in \mathbb{N}$)

- Prove that it is not possible that every vertex has degree 4 or more.
- Prove that G_n is 4-colourable, without using the four colour theorem.

Question A set of squares in the plane cuts the plane into several distinct regions. Prove that the regions can be coloured by two colours such that any two neighbouring regions receive different colours.

Drawings, Intersections,Crossings. Crossing Number.Basic Probability,Upper and Lower-Bounds.

Question What is the crossing number, $cr(G_n)$, of a graph G_n?

Question A graph on n vertices, G_n, has $7n$ edges. Give sharp lower bound on the crossing number in the form $cr(G_n) \geq \alpha n$, using the probabilistic method.

Question The complete bipartite graph with $n-n$ vertices in the two vertex classes is denoted by $K_{n,n}$. What is the magnitude of $cr(K_{n,n})$?

Point-Line incidences.Szemeredi-Trotter Theorem(bounds on incidences)

Question State the Szemeredi-Trotter Theorem. Sketch the proof using the crossing inequality.

Question What is the maximum number of $\sqrt{n}$-rich lines in an $n \times n$ integer grid?

The unit distances problem,distinct distances.

Erdos-SzekeresTheorem,Halving lines.

Question Draw a regular hexagon and add three extra points so that the set of 9 points doesn't contain a 7 points in convex position.

Question Draw 10 points in general position with at least 9 halving pairs.

Question State the cap-cup theorem.

Question Prove the identity $\binom{n-1}{k} + \binom{n-1}{k-1} = \binom{n}{k}$.

Circle Packing.Planar circle packing, Lattice Packing, Sphere Packing.

Question Given a planar lattice generated by two vectors $[2, 4], [-2, -2]$.

- Find two different vectors which generate the same lattice.
- Now we place a unit disk around every lattice point. Is this a packing? If so, then what is its density?

Question Given a space lattice generated by three vectors $[2, 4, 5], [-2, -2, -2], [3, -3, 3]$.

- Find three different vectors which generate the same lattice.
- Now we place a unit ball around every lattice point. Is this a packing? If so, then what is its density?

Question What is the largest convex subset of an $n \times n$ integer grid? Give an assymptotic answer.