MATH 223 HW 5.

**Question 1.** Give a formal proof to the following statement: If a row or a column of a square matrix consists of 0-s only then 0 is an eigenvalue of the matrix. Don’t use anything else but the definition of eigenvalue.

**Question 2.** A matrix $M$ is given as

$$
M = \begin{bmatrix}
2 & 1 & -1 \\
-1 & -3 & 0 \\
2 & 1 & -1
\end{bmatrix}.
$$

a, Find the eigenvalues and eigenvector of $M$.
b, Express vector $\vec{w} = [4, -3, 6]^t$ as the linear combination of the eigenvectors.
c, What are the eigenvectors and eigenvalues of $M^k$ for a given $k \in \mathbb{N}$?
d, Use your results above to find the vector $M^6 \cdot \vec{w}$ without calculating $M^6$.

**Question 3.** A matrix $M$ is defined as

$$
M = \begin{bmatrix}
1 & 2 & 1 & -1 \\
1 & -1 & 3 & 0 \\
3 & -2 & -1 & 4 \\
0 & 2 & 1 & -1
\end{bmatrix}.
$$

a, Calculate $\det(M)$ by cofactor expansion using the second column. Show your work step-by-step.
b, Find the reduced echelon form of $M$ using elementary row operations. Show your work step-by-step.

**Question 4.** Answer questions (a),(e) and (f) in Exercise 3.2.1 in Givental’s book.