Question 1. A linear transformation, \( T : \mathbb{R}^3 \to \mathbb{R}^3 \), is defined as \( T([1,1,1]^t) = [0,2,-1]^t \), \( T([0,2,-1]^t) = [1,1,1]^t \), and \( T([2,1,2]^t) = [1,-2,-1]^t \).

a, Find the transformation matrix.
b, Are there vectors, \( \vec{u} \), such that \( T(\vec{u}) \) is orthogonal to \( \vec{u} \)?
c, Are there vectors, \( \vec{u} \), such that \( T(\vec{u}) \) is parallel to \( \vec{u} \)?

Question 2. A transformation \( T : \mathbb{R}^3 \to \mathbb{R}^3 \) is defined as

\[
T(\vec{u}) = \begin{bmatrix}
1 & 2 & 1 \\
6 & -1 & 0 \\
-1 & -2 & -1
\end{bmatrix} \vec{u}.
\]

The eigenvalues of its matrix are 3, 0, and -4. Find the corresponding eigenvectors.

Question 3. A transformation, \( T : \mathbb{R}^2 \to \mathbb{R}^2 \), is defined as follows. First we reflect every point over the line \( \ell : 2x_1 - x_2 = 0 \), and then we rotate the points about the origin counterclockwise with angle \( \pi/3 \). Find all non-zero vectors which remain the same after the transformation, \( T(\vec{u}) = \vec{u} \).

Question 4. Find the eigenvalues and eigenvectors of the matrix

\[
\begin{bmatrix}
1 & -2 \\
-2 & 0
\end{bmatrix}.
\]

Question 4. Is there a value, \( \delta \), such that the two eigenvalues of the matrix below are 1 and -1?

\[
\begin{bmatrix}
\delta & \delta^3 \\
\delta^2 & -\delta
\end{bmatrix}.
\]