This assignment is due at the beginning of class on Friday, April 1.

1. Let $W$ be a standard Brownian motion with $W(0) = 0$. Let $0 < s < t$.
   
   (a) Determine the distribution of $W(s) + W(t)$.
   
   (b) Determine the conditional distribution of $W(s)$ given that $W(t) = b$. (Give the name of the distribution with the value of parameter(s).)

2. Let $\xi_i$ be i.i.d. with $P(\xi_i = 1) = P(\xi_i = -1) = \frac{1}{2}$. Let $X_0 = 0$ and $X_m = \xi_1 + \cdots + \xi_m$ for $m \geq 1$. For $n \geq 1$ and $t \in [0, 1]$, let $S_n(t) = n^{-1/2}X_{\lfloor nt \rfloor}$. For $k \geq 1$, let $0 < t_1 < \cdots < t_k \leq 1$. Let $W$ be a standard Brownian motion. Prove that, for each $k$, $(S_n(t_1), \ldots, S_n(t_k))$ converges in distribution to $(W(t_1), \ldots, W(t_k))$, as $n \to \infty$.


4. Durrett 8.1.2.

The following problems from Durrett are recommended for extra practice but are not to be handed in: 8.2.1, 8.2.2, 8.2.3.