1. (a) Durrett Exercise 5.7.4(i).
(b) Durrett Exercise 5.7.5.
(c) Consider the asymmetric random walk of part (b). Let $T_b$ be the hitting time for $b > 0$. Use the result of part (b) to show that $ET_1 = 1/(p - q)$. Conclude that $ET_b = b/(p - q)$.

2. Durrett Exercise 5.7.9.
The conclusion is immediate from the fact that the extinction probability is $\rho$ when $Z_0 = 1$, using independence of the family trees of the $x$ initial individuals. This problem gives a direct proof.

Recall Exercise 5.5.6 from previous assignment.

3. Let $Z_0, Z_1, \ldots$ be i.i.d. random variables with $P(Z_i = 1) = p$ and $P(Z_i = 0) = 1 - p$. Let $S_0 = 0, S_n = Z_1 + \cdots + Z_n$. In each of the following, determine whether $(X_n)_{n \geq 0}$ is a Markov chain. If it is, find its state space and transition matrix, and if it is not, give an example where $P(X_{n+1} = i | X_n = j, X_{n-1} = k)$ is not independent of $k$.
(a) $X_n = Z_n$
(b) $X_n = S_n$
(c) $X_n = S_0 + \cdots + S_n$.

4. Let $X = (X_n)_{n \geq 0}$ and $Y = (Y_n)_{n \geq 0}$ be independent symmetric simple random walks on $\mathbb{Z}$, with $X_0 = 0$ and $Y_0 = 2$. Show that $P(\exists n \text{ such that } X_n = Y_n) = 1$, by proving that $Z_n = X_n - Y_n$ is recurrent.

5. Let $X = (X_n)_{n \geq 0}$ and $Y = (Y_n)_{n \geq 0}$ be independent symmetric simple random walks in $\mathbb{Z}^d$, both started from the origin. Let

$$N = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} 1_{X_m = Y_n}$$

denote the number of intersections of the two walks. For $x \in \mathbb{Z}^d$, let $C(x) = \sum_{n=0}^{\infty} p_n(x)$, where $p_n(x)$ denotes the $n$-step transition probability.
(a) Show that $EN = \sum_{x \in \mathbb{Z}^d} C(x)^2$.
(b) Let $\phi_1(k) = E e^{ikX_1}$ denote the characteristic function of $X_1$. Show that

$$EN = \int_{[-\pi, \pi]^d} \frac{1}{[1 - \phi_1(k)]^2} \frac{dk}{(2\pi)^d},$$

and conclude that the expected number of intersections of the two walks is infinite in dimensions $d \leq 4$ and finite when $d > 4$. Hint: Recall the Parseval identity for the Fourier transform on $\mathbb{Z}^d$:

$$\sum_{x \in \mathbb{Z}^d} |f(x)|^2 = \int_{[-\pi, \pi]^d} |\hat{f}(k)|^2 \frac{dk}{(2\pi)^d}.$$

The following problems from Durrett are recommended for extra practice but are not to be handed in:
6.2.2, 6.2.3, 6.2.4, 6.2.6, 6.2.7.