Problems to hand in:

1. An astronomer is interested in measuring, in light years, the distance from her observatory to a distant star. Although she has a measuring technique, she knows that because of changing atmospheric conditions and experimental error, each time a measurement is made it will not yield the exact value but rather an approximate value. As a result the astronomer plans to make a series of measurements and then use the average of these as the estimated value of the actual distance. She believes that the values of the measurement errors are not systematic, so that the measurements are described by a random variable with mean \( d \) (the true distance) and a variance of 4 light years. Use the central limit theorem to determine approximately the number of measurements that should be made to be 95\% sure that the estimated distance is accurate to within ±0.5 light years.

2. Let \( X_i \) be i.i.d. Uniform(0,1) random variables, and let \( S_n = X_1 + \cdots + X_n \). Compute the characteristic function of \( X_i - \frac{1}{2} \) and use it compute the characteristic function of \( (S_n - n/2)/\sqrt{n} \). Using the continuity theorem, conclude that \( (S_n - n/2)/\sqrt{n} \) converges in distribution to an \( N(\mu, \sigma^2) \) random variable, and identify \( \mu \) and \( \sigma \). (The conclusion here is an immediate consequence of the central limit theorem, but this problem is asking you for a direct proof.)

3. Let \( X_1, X_2, \ldots \) be i.i.d. with \( X_i \geq 0, \text{EX}_i = 1, \text{Var}_X_i = \sigma^2 \in (0, \infty) \). Roughly, by the central limit theorem we might expect that
   \[
   2(\sqrt{S_n} - \sqrt{n}) = \int_{n}^{S_n} \frac{dx}{x^{1/2}} \approx \frac{S_n - n}{\sqrt{n}} \Rightarrow Z, 
   \]
   with \( Z \sim N(0,1) \).
   Give a complete proof that \( 2\sigma^{-1}(\sqrt{S_n} - \sqrt{n}) \Rightarrow Z \).

4. 11.4.2.

Problems not to be handed in:

9.5.2, 11.5.18.

For solutions, see: [http://www.probability.ca/jeff/grprobbook.html](http://www.probability.ca/jeff/grprobbook.html).